Practice Exam 04 by Richard X. Thripp: Chapters 10, 11, & 12 **Name:** Richard X. Thripp

[10.11, 10.39, 11.41, 11.46, 12.30, 12.44 : Solutions at daytonastate.org/physics] **Problems (10 points each):**

1. A solid, uniform cylinder with mass 7.50 kg and diameter 16.0 cm is spinning at 200 rpm on a thin, frictionless axle that passes along the cylinder axis. You design a simple friction brake to stop the cylinder by pressing the brake against the outer rim with a normal force. The coefficient of kinetic friction between the brake and rim is 0.25. What must the applied normal force be to bring the cylinder to rest after it has turned through 9.0 revolutions?

SOLVE for n

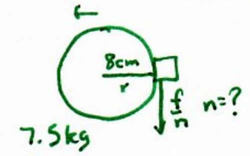
$I = \frac{1}{2}Mr^2$ $\quad f = \mu n,\ \tau = fr = I\alpha$

so $\alpha = \frac{fr}{I} = \frac{\mu n r}{I}$

$\frac{I}{\mu n r} = \frac{1}{\alpha} \Rightarrow$

$n = \frac{I\alpha}{\mu r}$

FIND α, angular stopping velocity



7.5 kg

$\omega = 200\text{rpm} = \frac{200 \cdot 2\pi}{60}\frac{rad}{sec} = 20.94\ rad/sec$

$\mu = 0.25 \quad \Delta\theta = 9\ rev = 18\pi\ rads$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \Rightarrow$

$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{0 - (20.94\frac{rad}{sec})^2}{2 \cdot 18\pi} = -3.88\frac{rad}{sec}$

$|\alpha| = 3.88\ rad/sec.$

$n = \frac{\frac{1}{2}(7.5kg)(0.08m)^2(3.88\frac{rad}{sec.})}{0.25 \cdot 0.08m} = \boxed{4.656N}$

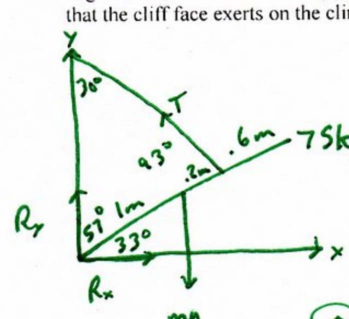
2. A star collapses from a radius of 8.0 * 10^5 km to a radius of 15 km, becoming a neutron star. The original star rotated once in 28 days. Assume the star is always a uniform, solid, rigid sphere. What is the angular speed of the neutron star?

$\omega_{orig} = \frac{1rev \times 2\pi}{28days \times 86{,}400sec} = 2.60 \times 10^{-6}\frac{rad}{sec}$

$\omega_{neutron} = 2.60 \times 10^{-6}\frac{rad}{sec}\left(\frac{8 \times 10^5 km}{15 km}\right) = \boxed{0.1385\frac{rad}{sec}}$

$\frac{0.1385\frac{rad}{sec} \times 86{,}400sec}{2\pi} = 1905\frac{rev}{day}$

3. A 75-kg mountain climber with a height of 1.80 m and a center of gravity 1.0 m from his feet rappels down a vertical cliff. His body is raised 33 degrees above the horizontal. He holds the rope 1.20 m from his feet, and it makes a 30 degree angle with the cliff face. Find (a) the tension his rope must support; (b) the horizontal and vertical components of the force that the cliff face exerts on the climber's feet.



$x: -T\sin 30° + R_x = 0$ ★

$y: T\cos 30° - mg + R_y = 0$

$\Sigma\tau = 0$

$-mg(1.0m)\cos 33° + T_x(1.2m)\cos 33° + T_y(1.2m)\sin 33° = 0$

$(-75kg)(9.8\frac{m}{s^2})(1m)(\cos 33°) + T(1.2m)(\cos 33°)(\cos 30°) + T(1.2m)(\sin 33°)(\sin 30°) = 0$

(a) $616.4N + 0.872T + 0.327T = 0$

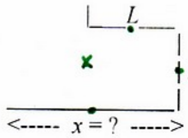
$1.199T = 616.4N$

$T \approx \boxed{514N}$

(b) ★ $R_x = T\sin 30° = 514N\sin 30° = \boxed{257N}$

$R_y = mg - T\cos 30° = (75kg)(9.8\frac{m}{s^2}) - 514N\cos 30° = \boxed{290N}$

4. A thin uniform metal rod is bent into three perpendicular segments, two of which have length L. You want to determine what the length of the third segment should be so that the unit will hang with two segments horizontal when it is supported by a hook. Find x in terms of L.



ρ = linear density = mass / length

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$0 = \frac{(\rho L)(\frac{L}{2}) + (\rho L)(L) + (\rho x)(L - \frac{x}{2})}{\rho(L + L + x)} = \frac{\frac{3}{2}L^2\rho + \rho x L - \frac{1}{2}\rho x^2}{2\rho L + \rho x}$$

$$= \frac{-x^2 + 2Lx + 3L^2}{4L + 2x} = \frac{x^2 - 2Lx - 3L^2}{-4L - 2x}$$

IF NUMERATOR IS ZERO, EXPRESSION IS ZERO

$$x^2 - 2Lx - 3L^2 = 0$$

$$x = \frac{-(-2L) \pm \sqrt{(-2L)^2 - 4(1)(-3L^2)}}{2(1)}$$

$$= \frac{2L \pm \sqrt{16L^2}}{2} = \frac{2L \pm 4L}{2}$$

only the positive root matters:

$$x = \frac{2L + 4L}{2} = \boxed{3L}$$

5. The International Space Station makes 15.75 revolutions per day in its orbit around the Earth. Assuming a circular orbit, how high is the satellite above the surface of the Earth?

MUST KNOW: KEPLER'S 3RD LAW, MASS OF EARTH, RADIUS OF EARTH,

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_e} \Rightarrow \frac{r^3}{T^2} = \frac{GM_e}{4\pi^2} \Rightarrow r = \sqrt[3]{\frac{GM_eT^2}{4\pi^2}}$$

$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M_e = 6 \times 10^{24}$ kg

$R_e = 6.4 \times 10^6$ m

$$T = \frac{86{,}400 \text{ sec.}}{15.75 \text{ rev.}} = 5486 \text{ sec.}$$

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \times 6 \times 10^{24} kg \times 5486^2 sec.}{4\pi^2}} = 6.73 \times 10^6 m$$

$$h = r - R_e = (6.73 \times 10^6 m) - (6.4 \times 10^6 m) = 3.3 \times 10^5 m$$

$$= \boxed{330 \text{ km}}$$

6. What is the mass of a black hole with diameter 1.1 * 10^-15 m?

MUST KNOW BLACK HOLE FORMULA:

$$c = \sqrt{\frac{2GM}{R_s}}$$

c = speed of light = $3 \times 10^8 \frac{m}{sec.}$

G = gravitational constant = $6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

R_s = Schwarzschild radius = radius of black hole ($\frac{1}{2}$ diameter)

SOLVE for M — Mass

$$c^2 = \frac{2GM}{R_s}$$

$$\frac{1}{c^2} = \frac{R_s}{2GM}$$

$$1 = \frac{R_s c^2}{2GM}$$

$$M = \frac{R_s c^2}{2G} = \frac{\frac{1}{2}(1.1 \times 10^{-15} m)(3 \times 10^8 \frac{m}{sec})^2}{2(6.67 \times 10^{-11} \frac{Nm^2}{kg^2})} = \boxed{3.71 \times 10^4 \text{ kg}}$$