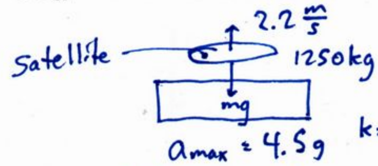
Practice Exam 03 by Richard X. Thripp: Chapters 7, 8, & 9 **Name:** Richard X. Thripp

[7.25, 7.53, 8.38, 8.58, 9.15, 9.34 : Solutions at daytonastate.org/physics] **Problems (10 points each):**

1. A spring must give a 1250-kg satellite a speed of 2.20 m/s relative to an orbiting space shuttle, applying a maximum acceleration of 4.50g. Ignore the spring's mass, the recoil kinetic energy of the shuttle, and changes in gravitational potential energy. What must the force constant of the spring be?



$a_{max} = 4.5g$

max accel.: 4.5 the weight (1250 kg)

$|F_s| = kx = ma_{max} \rightarrow k = \frac{ma_{max}}{x}$ = we need x

$\cancel{KE_i}^0 + U_i = KE_f + \cancel{U_f}^0$

$k = \frac{mv^2}{x^2} \leftarrow \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\left[x = \sqrt{\frac{mv^2}{k}}\right]$

$k = \frac{1250kg\,(4.5 \cdot 9.81)}{0.1096\,m} = \boxed{503478\ \frac{N}{m}}$

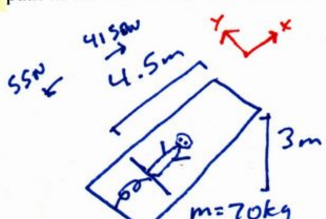
$\frac{ma_{max}}{x} = \frac{mv^2}{x^2}$ find 2 definition for k and set equal to each other

Completely unimportant

$\frac{ma_{max}}{x}x^2 = \frac{mv^2}{\cancel{x^2}} \rightarrow a_{max}x = v^2 \rightarrow x = \frac{v^2}{a_{max}} = \frac{(2.2\frac{m}{s})^2}{4.5 \cdot 9.81\frac{m}{s^2}} = \boxed{0.1096\,m}$

The spring is this compressed.

2. A 70-kg circus performer is shot from a spring cannon. The spring is of negligible mass and has a force constant of 900 N/m. The performer will compress the spring with a force of 4150 N. He will encounter an average friction force of 55 N on his 4.5 m path in the barrel. The end of the barrel is 3.0 m above his initial rest position. At what speed will he emerge?



$k = \cancel{1100}\ 900\ \frac{N}{m}$ $F = \cancel{4400}\ 4150\ N$

$|F_s| = kx \rightarrow x = \frac{|F_s|}{k} = \frac{4150\,N}{900\,\frac{N}{m}} = 4.6111\,m$

$fd\cos 180° = (\frac{1}{2}mv_f^2 + mgy_f) - (\cancel{\frac{1}{2}mv_i^2}^0 + \frac{1}{2}kx^2)$

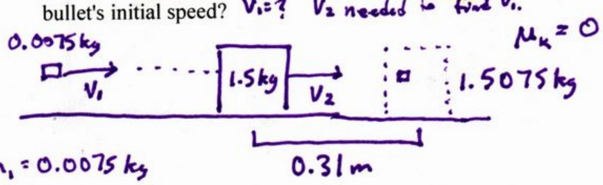
$-fd = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}kx^2$

$mv_f^2 = kx^2 - 2mgy_f - 2fd$

$v_f = \sqrt{\frac{kx^2 - 2mgy_f - 2fd}{m}} = \sqrt{\frac{900\frac{N}{m}(4.6111\,m)^2 - 2(70kg)(9.81\frac{m}{s^2})(3m) - 2(55N)(4.5\,m)}{70\,kg}}$

$= \sqrt{\frac{14{,}520.8189}{70}} = \sqrt{207.4403} = \boxed{14.4028\ \frac{m}{s}}$ → velocity is always in meters per second ($\frac{m}{s}$)

3. A 7.50-g bullet is fired horizontally into a 1.50-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.25. The bullet sticks in the block, and the block slides 0.310 m. What was the bullet's initial speed? $V_1 = ?$ V_2 needed to find V_1.



$\mu_k = 0.25$

$\vec{P}_i = \vec{P}_f$

$m_1V_1 + 0 = (m_1 + m_2)V_2$

$m_1 = 0.0075\,kg$

$m_2 = 1.5\,kg$

$V_2 = \frac{m_1V_1}{m_1 + m_2}$

Solve for V_1: $V_1 = \frac{V_2(m_1 + m_2)}{m_1} = \frac{1.2362\frac{m}{s}(1.5075kg)}{0.0075\,kg} = \boxed{248.4731\ \frac{m}{s}}$

$W_{nc} = E_f - E_i$

$-fd = (\cancel{\frac{1}{2}mv_f^2}^0 + \cancel{U_f}^0) - (\frac{1}{2}mV_i^2 + \cancel{U_i}^0)$

$fd = \frac{1}{2}m_2V_2^2 \rightarrow V_2 = \frac{m_1V_1}{m_1 + m_2}$

Using algebra

$\mu_k(m_1 + m_2)gd = \frac{1}{2}m_2V_2^2 \Rightarrow$

$V_2 = \sqrt{\frac{2\mu_k(m_1 + m_2)gd}{m_2}} = \sqrt{\frac{2\cdot(0.25\cdot 1.5075kg)(9.81\frac{m}{s^2}\cdot 0.31m)}{1.5\,kg}} = 1.2362\ \frac{m}{s}$

Find $\frac{dM}{dt}$!

4. A rocket is fired in deep space, where gravity is negligible. The rocket has an initial mass of $\overbrace{6500\text{ kg}}^{m}$, ejects gas at a relative velocity of magnitude $\underbrace{2200\text{ m/s}}_{V_e}$, and needs to reach an initial acceleration of $\underbrace{22.0\text{ m/s}^2}_{\vec{a}}$. How much gas must be ejected in the first second?

$\uparrow \vec{a} = 22\ \frac{m}{s^2} = \frac{dV}{dt}$

6500 kg

$V_e = 2200\ \frac{m}{s}$

$$M\frac{dV}{dt} = \left| V_e \frac{dM}{dt} \right| \Rightarrow \text{Solve for } \frac{dM}{dt}$$

$$\left|\frac{dM}{dt}\right| = \frac{M\frac{dV}{dt}}{V_e} = \frac{6500\text{kg}\cdot 22\frac{m}{s}}{2200\frac{m}{s}} = \boxed{65\text{ kg gas in 1st sec.}}$$

position → velocity → acceleration

Remember from calculus: $f(x)$ $f'(x)$ $f''(x)$

5. A flywheel is spinning at 650 rpm. Power is abruptly cut. After 25 unpowered seconds, the flywheel has made 250 complete revolutions. How many seconds after the power outage does the flywheel stop spinning?

650 rpm

$$\omega_0 = \frac{650\ \frac{rev.}{min.}}{60\text{ sec.}} = 10.8333\ \frac{rev}{sec}$$

$$\Delta\theta = \frac{1}{2}(\omega_f + \omega_0)t \Rightarrow$$

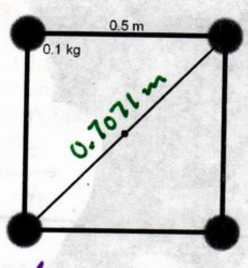
$$\omega_f = \frac{2\Delta\theta}{t} - \omega_0 \Rightarrow$$

$$\omega_f = \frac{2(250\text{ rev.})}{25\text{ sec.}} - 10.8333\ \frac{rev.}{sec} = 9.1667\ \frac{rev.}{sec}$$

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{9.1667\frac{rev}{sec} - 10.8333\frac{rev}{sec}}{25\text{ sec.}} = -0.0667\ \frac{rev}{sec^2}$$

$$t = \frac{\omega_f - \omega_0}{\alpha} + t_i = \frac{0\frac{rev}{sec} - 9.1667\frac{rev}{sec}}{-0.0667\frac{rev}{sec^2}} + 25\text{ sec.} = 137.4318\text{ sec} + 25\text{ sec.} = \boxed{162.4318\text{ sec.}}$$

6. Four spheres, 0.1 kg each, are arranged in a square 0.5 m on a side, connected by rods of negligible mass. Find the moment of inertia of the system about an axis (a) through the center of the square, perpendicular to its plane (coming out of the figure); I_1 (b) through the center of the square, horizontal to its plane, passing through two opposite spheres (as pictured). I_2



$$I_1 = 2(0.1\text{kg})(0.5\text{m})^2 = \boxed{0.05\text{ kg}\cdot\text{m}^2}$$

$$I_2 = 2m\frac{d^2}{2} = md^2 = 0.1\text{kg}\cdot(0.5\text{m})^2 = \boxed{0.025\text{ kg}\cdot\text{m}^2}$$

twice as hard as perpendicular turn ← perpendicular to plane = horizontal turn

half as hard as horizontal turn ← horizontal to plane = perpendicular turn

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