# Integration Review

1. $$\int \frac{2}{(t-9)^2}\,dt = 2\int u^{-2}\,du = 2\cdot\frac{u^{-1}}{-1} + C = \frac{-2}{u} + C = \boxed{\frac{-2}{t-9} + C}$$

$u = t-9$
$du = dt$

2. $$\int \left[v + \frac{1}{(3v-1)^3}\right] dv = \int v\,dv + \frac{1}{3}\int \frac{1\cdot 3}{(3v-1)^3}\,dv = \int v\,dv + \frac{1}{3}\int u^{-3}\,du = \frac{v^2}{2} + \frac{1}{3}\cdot\frac{u^{-2}}{-2} + C$$

$u = 3v-1$
$du = 3\,dv$

$$= \frac{v^2}{2} - \frac{1}{6u^2} + C = \boxed{\frac{v^2}{2} - \frac{1}{6(3v-1)^2} + C}$$

3. $$-\frac{1}{3}\int \frac{(t^2-3)(-3)}{-t^3+9t+1}\,dt = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = \boxed{-\frac{1}{3}\ln\left|-t^3+9t+1\right| + C}$$

$u = -t^3+9t+1$
$du = -3t^2+9\,dt$
$du = -3(t^2-3)\,dt$

4. $$-\frac{1}{4}\int x\sqrt{4-2x^2}\,dx\,(-4) = -\frac{1}{4}\int u^{\frac{1}{2}}\,du = -\frac{1}{\cancel{4}_{2}}\cdot\frac{\cancel{2}}{3}u^{\frac{3}{2}} + C = \boxed{-\frac{1}{6}(4-2x^2)^{\frac{3}{2}} + C}$$

$u = 4-2x^2$
$du = -4x\,dx$

5. $$\frac{1}{2}\int 5y\sqrt{1+2y}\,dy\,(2) = \frac{1}{2}\int 5\cdot\frac{(u-1)}{2}\cdot u^{\frac{1}{2}}\,du = \frac{5}{4}\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du = \frac{5}{4}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right] + C$$

$u = 1+2y \rightarrow \frac{u-1}{2} = y$
$du = 2dy$

$$= \frac{1}{2}u^{\frac{5}{2}} - \frac{5}{6}u^{\frac{3}{2}} + C$$

$$= \frac{1}{6}u^{\frac{3}{2}}\left[3u - 5\right] + C$$

$$= \boxed{\frac{1}{2}(1+2y)^{\frac{5}{2}} - \frac{5}{6}(1+2y)^{\frac{3}{2}} + C} \quad \text{OR} \quad \frac{1}{6}(1+2y)^{\frac{3}{2}}\left[3(1+2y) - 5\right] + C$$

$$= \frac{1}{6}(1+2y)^{\frac{3}{2}}\left[6y-2\right] + C$$

$$= \boxed{\frac{1}{3}(1+2y)^{\frac{3}{2}}\left[3y-1\right] + C}$$

6. $$\int \tan^2 x\,dx = \int(\sec^2 x - 1)\,dx = \boxed{\tan x - x + C}$$

$1 + \tan^2 x = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$

7. $\int \sin^5 x \cos x\, dx = \int u^5\, du = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}\sin^6 x + C}$

$u = \sin x$

$du = \cos x\, dx$

8. $\int \frac{2\pi}{3x^2}\, dx = \frac{2\pi}{3}\int x^{-2}\, dx = \frac{2\pi}{3} \cdot \frac{x^{-1}}{-1} + C = \boxed{\frac{-2\pi}{3x} + C}$

9. $\frac{1}{3}\int \sec(3x)\tan(3x)\, dx \cdot 3 = \frac{1}{3}\int \sec u \tan u\, du = \frac{1}{3}\sec u + C = \boxed{\frac{1}{3}\sec(3x) + C}$

$u = 3x$

$du = 3dx$

10. $\int \frac{1-3x}{\sqrt{x+3}}\, dx = \int \frac{1-3(u-3)}{\sqrt{u}}\, du = \int \frac{1-3u+9}{\sqrt{u}}\, du = \int \frac{10-3u}{\sqrt{u}}\, du = \int \left(10u^{-\frac{1}{2}} - 3u^{\frac{1}{2}}\right) du$

$u = x+3 \longrightarrow u-3 = x$

$du = dx$

$= 10 \cdot 2u^{\frac{1}{2}} - 3 \cdot \frac{2}{3}u^{\frac{3}{2}} + C = 20u^{\frac{1}{2}} - 2u^{\frac{3}{2}} + C = 2u^{\frac{1}{2}}(10-u) + C$

$= \boxed{20(x+3)^{\frac{1}{2}} - 2(x+3)^{\frac{3}{2}} + C}$ OR $2(x+3)^{\frac{1}{2}}(10-(x+3)) + C$

$= \boxed{2(x+3)^{\frac{1}{2}}(7-x) + C}$

11. $\int \frac{5y^2+2y}{\sqrt{y}}\, dy = \int 5y^{\frac{3}{2}} + 2y^{\frac{1}{2}}\, dy = 5 \cdot \frac{2}{5}y^{\frac{5}{2}} + 2 \cdot \frac{2}{3}y^{\frac{3}{2}} + C = \boxed{2y^{\frac{5}{2}} + \frac{4}{3}y^{\frac{3}{2}} + C}$

OR $\boxed{\frac{2}{3}y^{\frac{3}{2}}(3y+2) + C}$

12. $\int \left[\frac{1}{3x-1} - \frac{1}{3x+1}\right] dx = \frac{1}{3}\int \frac{1}{3x-1}3\cdot dx - \frac{1}{3}\int \frac{1}{3x+1}3\cdot dx = \frac{1}{3}\int \frac{1}{u}\, du - \frac{1}{3}\int \frac{1}{v}\, dv$

$u = 3x-1$ $\quad$ $v = 3x+1$

$du = 3dx$ $\quad$ $dv = 3dx$

$= \frac{1}{3}\ln|u| - \frac{1}{3}\ln|v| + C = \boxed{\frac{1}{3}\ln|3x-1| - \frac{1}{3}\ln|3x+1| + C}$

13. $-\int \frac{-1}{\sqrt{x}(1-2\sqrt{x})}\, dx = -\int \frac{1}{u}\, du = -\ln|u| + C = \boxed{-\ln|1-2\sqrt{x}| + C}$

$u = 1-2\sqrt{x}$

$du = -2 \cdot \frac{1}{2}x^{-\frac{1}{2}}\, dx$

$du = -\frac{1}{\sqrt{x}}\, dx$

14. $\displaystyle\int \frac{1+\sin x}{\cos x}\,dx = \int \frac{(1+\sin x)(1-\sin x)}{\cos x\,(1-\sin x)}\,dx = \int \frac{1-\sin^2 x}{\cos x\,(1-\sin x)}\,dx = \int \frac{\cos^2 x}{\cos x\,(1-\sin x)}\,dx$

$\displaystyle = -\int \frac{-\cos x}{1-\sin x}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = \boxed{-\ln|1-\sin x| + C}$

$u = 1-\sin x$

$du = -\cos x\,dx$

OR: $\displaystyle\int \frac{1+\sin x}{\cos x}\,dx = \int (\sec x + \tan x)\,dx = \int \sec x\,dx + \int \tan x\,dx = \int \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x}\,dx + -\int \frac{-\sin x}{\cos x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$\displaystyle = \int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x}\,dx - \int \frac{1}{u}\,du = \int \frac{1}{v}\,dv - \int \frac{1}{u}\,du = \ln|v| - \ln|u| + C$

$v = \sec x + \tan x$

$dv = \sec x\tan x + \sec^2 x\,dx$

$= \boxed{\ln|\sec x+\tan x| - \ln|\cos x| + C}$

OR $\boxed{\ln|\sec x+\tan x| + \ln|\sec x| + C}$

15. $\displaystyle -\frac{1}{2}\int (-2x+5)^{\frac{2}{3}}\,dx\,(-2) = -\frac{1}{2}\int u^{\frac{2}{3}}\,du = -\frac{1}{2}\cdot\frac{3}{5}u^{\frac{5}{3}} + C = \boxed{-\frac{3}{10}(-2x+5)^{\frac{5}{3}} + C}$

$u = -2x+5$

$du = -2dx$

16. $\displaystyle\int \frac{2x}{x^2\sqrt{x^4-1}}\,dx = \int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1} u + C = \boxed{\sec^{-1}(x^2) + C}$

see page 213

$u = x^2$

$du = 2x\,dx$

17. $\displaystyle\int \frac{1}{(1+x^2)\tan^{-1} x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\tan^{-1} x| + C}$

$u = \tan^{-1} x$

$du = \frac{1}{1+x^2}\,dx$

18. $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x\,dx}{1+(e^x)^2} = \int \frac{1}{1+u^2}\,du = \tan^{-1} u + C = \boxed{\tan^{-1}(e^x) + C}$

$u = e^x$

$du = e^x\,dx$

19. $\displaystyle\int \frac{2\cos x}{1+\sin^2 x}\,dx = 2\int \frac{1}{1+u^2}\,du = 2\tan^{-1}u + C = \boxed{2\tan^{-1}(\sin x) + C}$

$u = \sin x$

$du = \cos x\,dx$

20. $\displaystyle\int \frac{\tan(\ln x)}{x}\,dx = \int \tan u\,du = -\ln|\cos u| + C$ OR $\ln|\sec u| + C$

$= \boxed{-\ln|\cos(\ln x)| + C}$ OR $\boxed{\ln|\sec(\ln x)| + C}$

$u = \ln x$

$du = \frac{1}{x}\,dx$