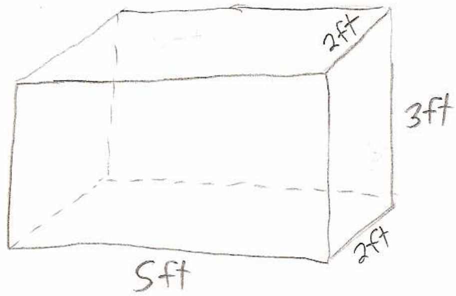
8.3 homework

1.



a) Find the hydrostatic pressure on the bottom of the aquarium.

$P = \delta d = 62.5 \frac{lb}{ft^3} \cdot 3ft = \boxed{187.5 \frac{lb}{ft^2}}$

$\boxed{\text{Formula 1 P. 540}}$

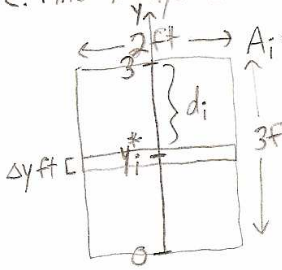
b. Find the hydrostatic force on the bottom.

$F = PA = 187.5 \frac{lb}{ft^2} \cdot 10 ft^2 = \boxed{1875 \text{ lb.}}$

Formula 0 P. 540

Area of bottom

c. Find the hydrostatic force on one end of the aquarium.



A_i = Area of ith rectangular strip $= (2ft)\cdot(\Delta y\, ft) = 2\Delta y\ ft^2$

3ft

P_i = Pressure on the ith strip $= \delta d_i = 62.5 \frac{lb}{ft^3} \cdot (3-y_i^*)\, ft = 62.5(3-y_i^*) \frac{lb}{ft^2}$

depth of the ith strip below water's surface

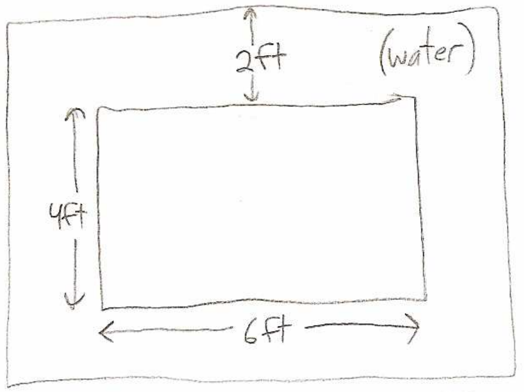
Formula 0 P. 540

$F_i = P_i A_i = 62.5(3-y_i^*) \frac{lb}{ft^2} \cdot 2\Delta y\, ft^2 = 125(3-y_i^*)\Delta y$ lb.

F = total force on the end of the aquarium $= \lim_{n\to\infty} \sum_{i=1}^{n} 125(3-y_i^*)\Delta y = \int_0^3 125(3-y)\,dy$

$= 125\left[3y - \frac{y^2}{2}\right]_0^3 = 125\left[9 - \frac{9}{2}\right] = 125 \cdot \frac{9}{2} = 562.5$ lb.

3. Find the hydrostatic force against one side of the plate.



A_i = Area of ith rectangular strip $= (6\text{ ft})(\Delta y\text{ ft})$

$= 6\Delta y \text{ ft}^2$

P_i = Pressure on the ith strip $= \delta d_i = 62.5 \frac{\text{lb}}{\text{ft}^3} \cdot (6 - y_i^*)\text{ ft}$

Formula 1 p.540

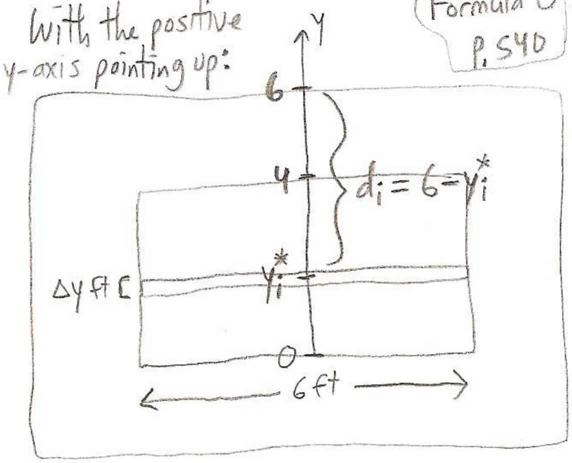
$= 62.5(6 - y_i^*) \frac{\text{lb}}{\text{ft}^2}$.

$F_i = P_i A_i = 62.5(6 - y_i^*) \frac{\text{lb}}{\cancel{\text{ft}^2}} \cdot 6\Delta y \cancel{\text{ft}^2}$

Formula 0 p.540

$= 375(6 - y_i^*)\Delta y$ lb.

With the positive y-axis pointing up:



F = total force on one side of the plate

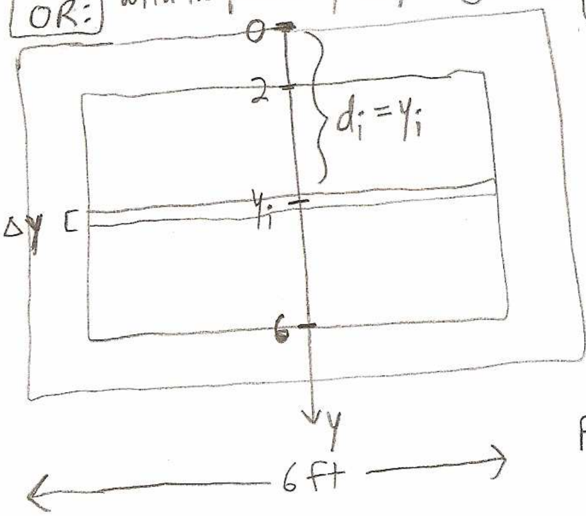
$= \lim_{n\to\infty} \sum_{i=1}^{n} 375(6 - y_i^*)\Delta y$

$= \int_0^4 375(6-y)\,dy$

$= 375\left[6y - \frac{y^2}{2}\right]_0^4$

$= 375[24 - 8] = 375 \cdot 16 = \boxed{6000 \text{ lb.}}$

$\boxed{\text{OR:}}$ with the positive y axis pointing down:



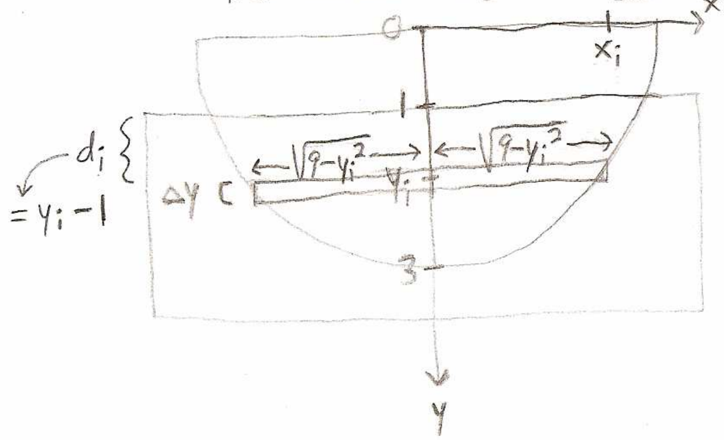
$A_i = 6\Delta y, \quad P_i = \delta d_i = 62.5 y_i$

$F_i = P_i A_i = (62.5 y_i)(6\Delta y) = 375\, y_i\, \Delta y$

$F = \lim_{n\to\infty} \sum_{i=1}^{n} F_i = \int_2^6 375 y\,dy = 375\left[\frac{y^2}{2}\right]_2^6$

$= \frac{375}{2}[36 - 4] = \frac{375}{2} \cdot 32 = \boxed{6000 \text{ lb.}}$

5. Consider the positive y direction to be down with the origin at the circle's center:



Equation of the full circle: $x^2 + y^2 = 9$

Solving for x: $x = \pm\sqrt{9-y^2}$

A_i = area of ith strip $= 2\sqrt{9-y_i^2}\,\Delta y \ m^2$

P_i = pressure on ith strip $= \rho g d_i$

$= 1000\,\frac{kg}{m^3}\cdot 9.8\,\frac{m}{s^2}\cdot (y_i - 1)\,m = 9800(y_i-1)$ Pa. ← Pascals

$= 9800y_i - 9800$ Pa. ← OR $\frac{N}{m^2}$

$$F_i = P_iA_i = (9800y_i - 9800)\cdot 2\sqrt{9-y_i^2}\,\Delta y = 19{,}600\,y_i\sqrt{9-y_i^2}\,\Delta y - 19{,}600\sqrt{9-y_i^2}\,\Delta y$$

total force → $F = \lim_{n\to\infty}\sum_{i=1}^{n}\left[19{,}600\,y_i\sqrt{9-y_i^2}\,\Delta y - 19{,}600\sqrt{9-y_i^2}\,\Delta y\right]$

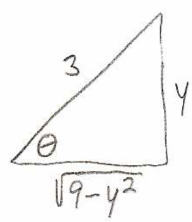
$$= \left(-\tfrac{1}{2}\right)\int_1^3 19{,}600\,y\sqrt{9-y^2}\,(-2)\,dy - \int_1^3 19{,}600\sqrt{9-y^2}\,dy$$

$u = 9-y^2,\ du = -2y\,dy \qquad y = 3\sin\theta,\ dy = 3\cos\theta\,d\theta$

y	u
1	8
3	0

$$= -9800\int_8^0 u^{\frac{1}{2}}\,du - 19{,}600\int \underbrace{3\cos\theta\cdot 3\cos\theta}_{9\cos^2\theta}\,d\theta$$

$$= 9800\int_0^8 u^{\frac{1}{2}}\,du - 176{,}400\cdot\tfrac{1}{2}\int(1+\cos 2\theta)\,d\theta$$



$$= 9800\cdot\tfrac{2}{3}\left[u^{\frac{3}{2}}\right]_0^8 - 88{,}200\left[\theta + \frac{\overbrace{\sin 2\theta}^{\cancel{2}\sin\theta\cos\theta}}{\cancel{2}}\right]$$

$$= \frac{19{,}600}{3}\cdot 8^{\frac{3}{2}} - 88{,}200\left[\sin^{-1}\frac{y}{3} + \frac{y}{3}\cdot\frac{\sqrt{9-y^2}}{3}\right]_1^3$$

$$= \frac{19{,}600}{3}\cdot(2\sqrt{2})^3 - 88{,}200\left[(\sin^{-1}1 + 0) - \left(\sin^{-1}\frac{1}{3} + \frac{2\sqrt{2}}{9}\right)\right]$$

$$= \frac{19{,}600}{3}\cdot 16\sqrt{2} - 88{,}200\cdot\frac{\pi}{2} + 88{,}200\sin^{-1}\frac{1}{3} + 19{,}600\sqrt{2}$$

$$= \boxed{\frac{313{,}600\sqrt{2}}{3} - 44{,}100\pi + 88{,}200\sin^{-1}\frac{1}{3} + 19{,}600\sqrt{2}\ N}$$ ← Newtons

$$\approx \boxed{66{,}980\ N}$$

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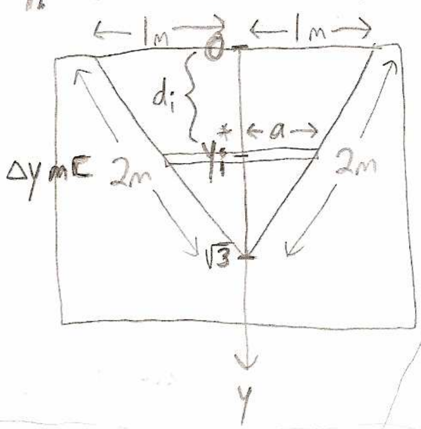
7.



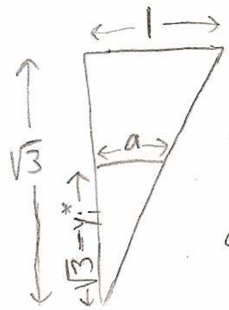
Consider the positive y axis to point down.

A_i = area of ith strip $= 2a\,\Delta y\ m^2 = 2\left[\dfrac{\sqrt{3}-y_i}{\sqrt{3}}\right]\Delta y\ m^2$

$= \left[2 - \dfrac{2}{\sqrt{3}}y_i\right]\Delta y\ m^2$

P_i = pressure on ith strip $= \rho g d_i = 1000\dfrac{kg}{m^3}\cdot 9.8\dfrac{m}{s^2}\cdot y_i\ m$

$= 9800\,y_i\ Pa\ \left(\text{OR } \dfrac{N}{m^2}\right)$



By similar triangles,

$\dfrac{1}{\sqrt{3}} = \dfrac{a}{\sqrt{3}-y_i^*}$

$a = \dfrac{\sqrt{3}-y_i^*}{\sqrt{3}}$

$F_i = P_i A_i = 9800 y_i\left[2 - \frac{2}{\sqrt{3}}y_i\right]\Delta y$

$= 9800\left[2y_i - \frac{2}{\sqrt{3}}y_i^2\right]\Delta y$

total force $\rightarrow F = \lim_{n\to\infty}\sum_{i=1}^{n} 9800\left[2y_i - \frac{2}{\sqrt{3}}y_i^2\right]\Delta y = \int_0^{\sqrt{3}} 9800\left[2y - \frac{2}{\sqrt{3}}y^2\right]dy$

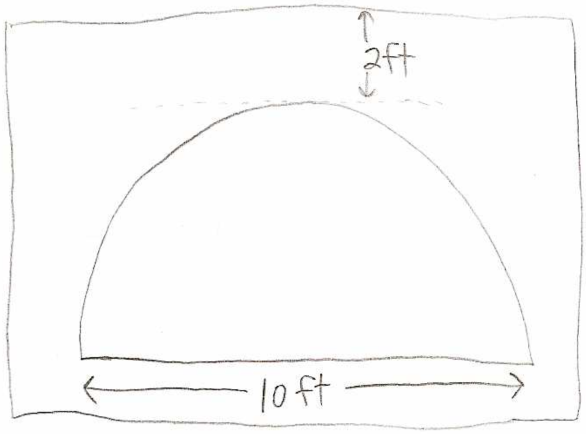
$= 9800\left[y^2 - \frac{2}{\sqrt{3}}\frac{y^3}{3}\right]_0^{\sqrt{3}}$

$= 9800\left[\left(3 - \frac{2}{\cancel{\sqrt{3}}}\cdot\frac{\cancel{3}\cancel{\sqrt{3}}}{\cancel{3}}\right) - (0-0)\right]$

$= 9800\left[3-2\right]$

$= \boxed{9800\ N}$

9. Find the hydrostatic force against one side of the plate.



Equation of the circle: $x^2+y^2=25$.

Upper half only: $x=\pm\sqrt{25-y^2}$

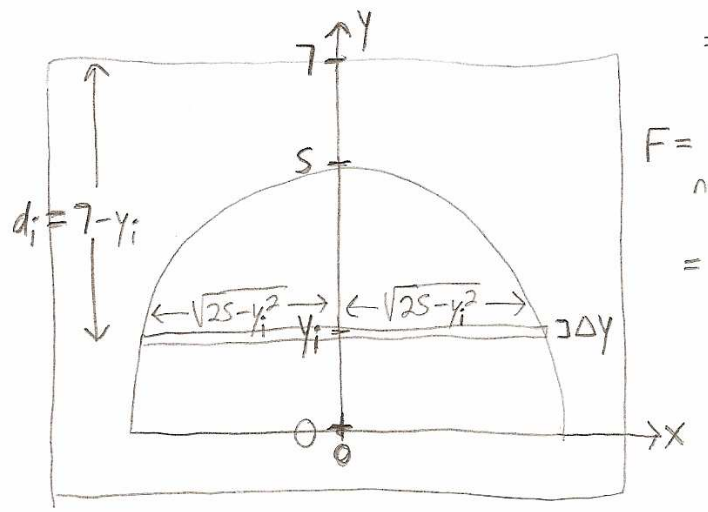
A_i = area of ith strip = $2\sqrt{25-y_i^2}\,\Delta y \text{ ft}^2$.

P_i = pressure on ith strip = δd_i

$= 62.5\,\frac{\text{lb}}{\text{ft}^3}\cdot(7-y_i)\text{ ft} = 62.5(7-y_i)\,\frac{\text{lb}}{\text{ft}^2}$.

$F_i = P_iA_i = 125(7-y_i)\sqrt{25-y_i^2}\,\Delta y \text{ lb}$

$= 875\sqrt{25-y_i^2}\,\Delta y - 125y_i\sqrt{25-y_i^2}\,\Delta y$

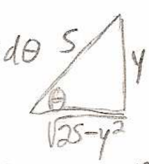


$F = \lim_{n\to\infty}\sum_{i=1}^{n} F_i$ See Note at bottom.

$= \int_0^5 875\sqrt{25-y^2}\,dy - \left(\tfrac{-1}{2}\right)\int_0^5 125y\sqrt{25-y^2}\,dy\,(-2)$

$y = 5\sin\theta$

$dy = 5\cos\theta\,d\theta$



$u = 25-y^2$

$du = -2y\,dy$

y	u
0	25
5	0

$= 875\int 25\cos^2\theta\,d\theta + \frac{125}{2}\int_{25}^{0} u^{\frac{1}{2}}\,du = 21{,}875\cdot\frac{1}{2}\int[1+\cos 2\theta]\,d\theta + \frac{125}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\left[u^{\frac{3}{2}}\right]_{25}^{0}$

$= \frac{21{,}875}{2}\left[\theta + \frac{\overset{2\sin\theta\cos\theta}{\sin 2\theta}}{2}\right] + \frac{125}{3}\left[0-125\right] = \frac{21{,}875}{2}\left[\sin^{-1}\frac{y}{5} + \frac{y}{5}\cdot\frac{\sqrt{25-y^2}}{5}\right]_0^5 - \frac{15{,}625}{3}$

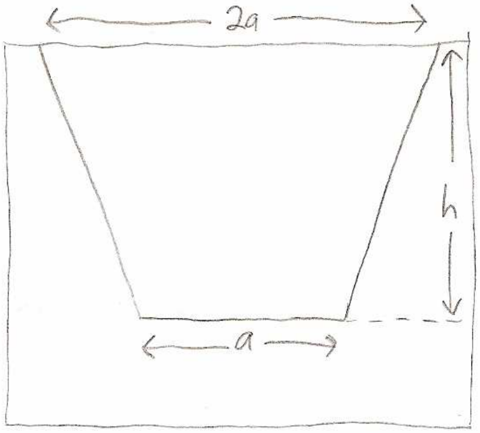
$= \frac{21{,}875}{2}\left[\left(\frac{\pi}{2}+0\right)-(0+0)\right] - \frac{15{,}625}{3} = \frac{21{,}875\pi}{4} - \frac{15{,}625}{3}$

$\approx \boxed{11{,}972 \text{ lb}}$

* Note: $\int_0^5\sqrt{25-y^2}\,dy$ can be thought of as the area of $\frac{1}{4}$ of a circle with radius 5: $\frac{1}{4}\cdot(\pi\cdot 5^2) = \frac{25\pi}{4}$.

So, $875\int_0^5\sqrt{25-y^2}\,dy = 875\cdot\frac{25\pi}{4} = \frac{21{,}875\pi}{4}$. This can avoid doing the trig substitution.

11. Find the hydrostatic force against one side of the plate.

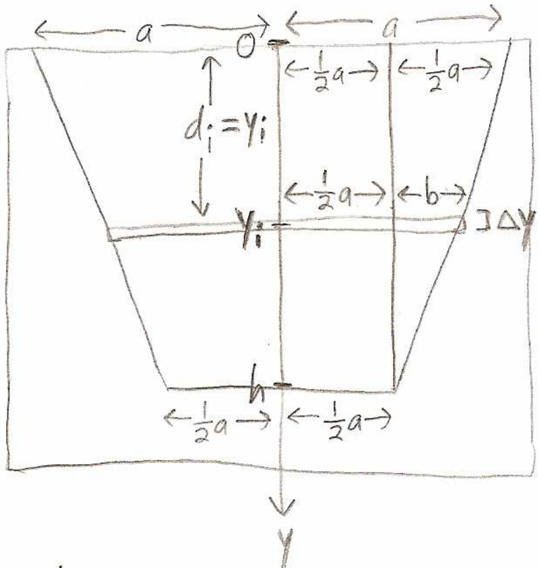


A_i = area of ith strip $= (a+2b)\Delta y = \left[a + \cancel{2}\left[\dfrac{a(h-y_i)}{\cancel{2}h}\right]\right]\Delta y$

$= \left[a + \frac{a}{h}(h-y_i)\right]\Delta y = \left[a + a - \frac{a}{h}y_i\right]\Delta y = \left[2a - \frac{a}{h}y_i\right]\Delta y$

$P_i = \rho g d_i = \rho g y_i.$

I'll keep ρg until the end, since we are not given units.



$F_i = P_i A_i = \rho g y_i \left(2a - \frac{a}{h}y_i\right)\Delta y$

$= \rho g \left(2a y_i - \frac{a}{h}y_i^2\right)\Delta y$

$F = \lim_{n\to\infty} \sum_{i=1}^{n} F_i = \int_0^h \rho g \left(2ay - \frac{a}{h}y^2\right) dy$

$= \rho g \left[ay^2 - \frac{a}{h}\cdot\frac{y^3}{3}\right]_0^h$

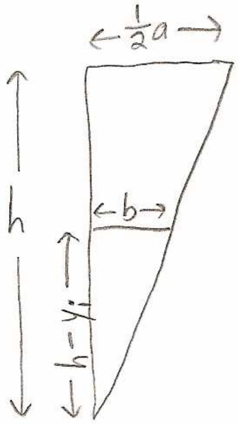
$= \rho g \left[\left(ah^2 - \frac{a}{h}\cdot\frac{h^3}{3}\right) - (0-0)\right]$

$= \rho g \left[ah^2 - \frac{1}{3}ah^2\right]$

$= \boxed{\rho g \left[\frac{2}{3}ah^2\right]}$

$= 1000\frac{kg}{m^3}\cdot 9.8\frac{m}{s^2}\cdot\frac{2}{3}ah^2\, m^3$ OR $62.5\frac{lb}{ft^3}\cdot\frac{2}{3}ah^2\, ft^3$

$= \boxed{\frac{19{,}600}{3}ah^2\ N}$ OR $\boxed{\frac{125}{3}ah^2\ lb}$



Similar triangles:

$\dfrac{\frac{1}{2}a}{h} = \dfrac{b}{h-y_i}$

$b = \dfrac{\frac{1}{\cancel{2}}a(h-y_i)\cdot\cancel{2}}{h\cdot 2}$

$b = \dfrac{a(h-y_i)}{2h}$

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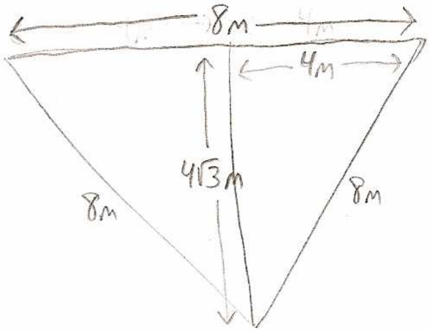
13. Find the hydrostatic force on the end of the trough shown.

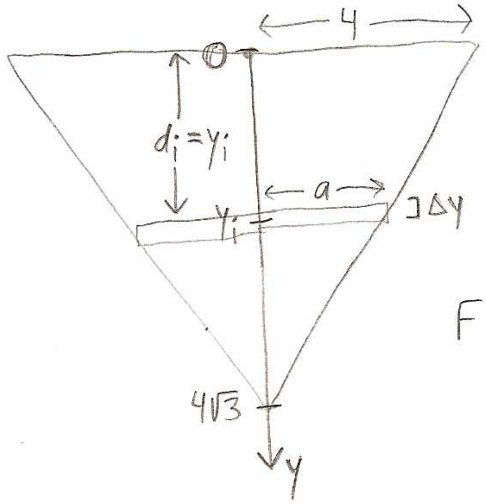


A_i = area of ith strip = $2a\,\Delta y\ m^2 = 2\left[4 - \frac{\sqrt{3}}{3}y_i\right]\Delta y\ m^2$

$= \left[8 - \frac{2\sqrt{3}}{3}y_i\right]\Delta y\ m^2.$

P_i = pressure on ith strip = $\rho g d_i$

$= 840\,\frac{kg}{m^3}\cdot 9.8\,\frac{m}{s^2}\cdot y_i\,m = 8232\,y_i\ Pa\ \left(\text{OR } \frac{N}{m^2}\right).$



$F_i = P_iA_i = 8232y_i\left[8 - \frac{2\sqrt{3}}{3}y_i\right]\Delta y\ N$

$= 8232\left[8y_i - \frac{2\sqrt{3}}{3}y_i^2\right]\Delta y\ N.$

$F = \lim_{n\to\infty}\sum_{i=1}^{n} F_i = \int_0^{4\sqrt{3}} 8232\left[8y - \frac{2\sqrt{3}}{3}y^2\right]dy$

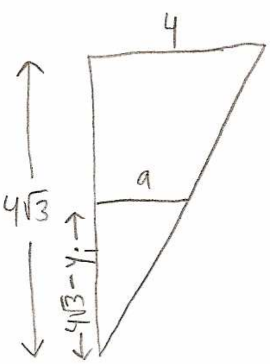
$= 8232\left[4y^2 - \frac{2\sqrt{3}}{3}\cdot\frac{y^3}{3}\right]_0^{4\sqrt{3}}$

$= 8232\left[4\cdot 48 - \frac{2\sqrt{3}}{9}\cdot 64\cdot 3\sqrt{3}\right]$

$= 8232\left[192 - 128\right]$

$= 8232\cdot 64$

$= \boxed{526{,}848\ N}$



Similar triangles:

$\frac{4}{4\sqrt{3}} = \frac{a}{4\sqrt{3} - y_i}$

$a = \frac{\cancel{4}(4\sqrt{3} - y_i)}{\cancel{4}\sqrt{3}}$

$a = \frac{4\sqrt{3} - y_i}{\sqrt{3}}$

$a = 4 - \frac{1}{\sqrt{3}}y_i$

$a = 4 - \frac{\sqrt{3}}{3}y_i$

8.3 Exs for class Hydrostatic force (Fluid force)

$P = \frac{F}{A}$ so $\boxed{F = PA}$ Formula ① P.540 F = force P = pressure A = area

Formula ② P.540 → $P = \underbrace{\rho g d}_{\text{SI units}} = \underbrace{\delta d}_{\text{US units}}$

$\rho = \frac{1000\,kg}{m^3}$ = density of water

"rho" ← Greek letter (lowercase rho)

$g = 9.8\frac{m}{s^2}$

$\delta = 62.5\frac{lb}{ft^3}$ weight density of water

$P = \underbrace{\rho g d}_{\text{SI units (metric)}}$ OR $P = \underbrace{\delta d}_{\text{U.S. Customary units}}$

$\rho = 1000\frac{kg}{m^3}$ ← for water, $g = 9.8\frac{m}{s^2}$; $\delta = 62.5\frac{lb}{ft^3}$ ← for water

SI units: $A = m^2$, $P = \frac{N}{m^2}$ = Pa (Pascals), $F = N$. (newtons) (N ← Newtons)

US units: $A = ft^2$, $P = \frac{lb}{ft^2}$, $F = lb$

Steps for calculating the hydrostatic force on a vertical plate submerged in fluid:

① Decide how to orient the positive direction of the y axis. (Pointing UP or Down)

② Get A_i = area of ith rectangular strip. (Sometimes similar Δs is necessary.)

③ Get P_i = pressure on ith rect. strip $P_i = \underbrace{\rho g}_{\substack{9800 \\ \text{for water}}} d_i$ OR $\underbrace{\delta}_{\substack{62.5 \\ \text{for water}}} d_i$

④ Get F_i = force on ith strip = $P_i A_i$

⑤ F = total force = $\lim_{n\to\infty} \sum_{i=1}^{n} F_i$ = an integral.