7.6 homework

$\frac{u}{\sqrt{2}}=x \Rightarrow \frac{u^2}{2}=x^2$

1. $\int \frac{\sqrt{7-2x^2}}{x^2}\,dx$ $\quad a=\sqrt{7}\quad u=\sqrt{2}x \quad du=\sqrt{2}\,dx \quad \frac{du}{\sqrt{2}}=dx$

$$= \int \frac{\sqrt{a^2-u^2}}{\left(\frac{u^2}{2}\right)}\cdot\frac{du}{\sqrt{2}} = \frac{2}{\sqrt{2}}\int\frac{\sqrt{a^2-u^2}}{u^2}\,du$$

$$= \sqrt{2}\int\frac{\sqrt{a^2-u^2}}{u^2}\,du$$

$\boxed{33}$ $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2}-\sin^{-1}\frac{u}{a}+C$

$$\therefore = \sqrt{2}\left[-\frac{1}{\sqrt{2}x}\sqrt{7-2x^2}-\sin^{-1}\frac{\sqrt{2}x}{\sqrt{7}}\right]+C$$

$$= \boxed{-\frac{1}{x}\sqrt{7-2x^2}-\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{7}}\cdot x\right)+C}$$

3. $\frac{1}{\pi}\int \sec^3(\pi x)\,dx^{(\pi)} = \frac{1}{\pi}\int\sec^3 u\,du \overset{\boxed{71}}{\overset{\downarrow}{=}} \frac{1}{\pi}\cdot\frac{1}{2}\left[\sec u\tan u+\ln\left|\sec u+\tan u\right|\right]+C$

$u=\pi x$

$du=\pi\,dx$

$$= \frac{1}{2\pi}\left[\sec\pi x\tan\pi x+\ln\left|\sec\pi x+\tan\pi x\right|\right]+C$$

without the table entry →

5. $\int_0^1 2x\cos^{-1}x\,dx = \left[x^2\cos^{-1}x\right]_0^1 + \int_0^1\frac{x^2}{\sqrt{1-x^2}}\,dx = \left(1\cos^{-1}1-0\right)+\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}\,d\theta$

$u=\cos^{-1}x \qquad v=x^2$

$du=\frac{-1}{\sqrt{1-x^2}}\,dx \qquad dv=2x\,dx$

$\cancel{u=\sqrt{1-x^2}}$

$\cancel{u^2=1-x^2}$

$\cancel{x^2=1-u^2}$

$\cancel{2x\,dx=-2u\,du}$

$x=\sin\theta$

$dx=\cos\theta\,d\theta$

x	θ
0	0
1	$\frac{\pi}{2}$

$$= (1\cdot 0-0)+\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(1-\cos 2\theta\right)d\theta = \frac{1}{2}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right)-\left(0-0\right)\right] = \boxed{\frac{\pi}{4}}$$

OR

using table entry 91 →

5. $\int_0^1 2x\cos^{-1}x\,dx = 2\left[\frac{2x^2-1}{4}\cos^{-1}x-\frac{x\sqrt{1-x^2}}{4}\right]_0^1 = 2\left[\left(\frac{1}{4}\cos^{-1}1-0\right)-\left(-\frac{1}{4}\cos^{-1}0-0\right)\right]$

$$= 2\left[\cancel{\left(\frac{1}{4}\cdot 0\right)}-\left(-\frac{1}{4}\cdot\frac{\pi}{2}\right)\right]$$

$\boxed{91}$ $\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u-\frac{u\sqrt{1-u^2}}{4}$

$$= 2\cdot\frac{\pi}{8} = \boxed{\frac{\pi}{4}}$$

without table →

$$7.\ \frac{1}{\pi}\int \tan^3(\pi x)\,dx\,(\pi) = \frac{1}{\pi}\int \tan^3 u\,du = \frac{1}{\pi}\int (\sec^2 u - 1)\tan u\,du =$$

$u = \pi x$

$du = \pi dx$

$$= \frac{1}{\pi}\int \tan u \sec^2 u\,du - \frac{1}{\pi}\int \tan u\,du = \boxed{\frac{1}{\pi}\frac{\tan^2 u}{2} + \frac{1}{\pi}\ln|\cos u| + C}$$

$$= \boxed{\frac{1}{2\pi}\tan^2(\pi x) + \frac{1}{\pi}\ln|\cos \pi x| + C.}$$

OR

with table entry 69 →

$$7.\ \frac{1}{\pi}\int \tan^3(\pi x)\,dx\,(\pi) = \frac{1}{\pi}\int \tan^3 u\,du \overset{\boxed{69}}{=} \frac{1}{\pi}\left[\frac{1}{2}\tan^2 u + \ln|\cos u|\right] + C$$

$u = \pi x$

$du = \pi dx$

$$= \boxed{\frac{1}{2\pi}\tan^2(\pi x) + \frac{1}{\pi}\ln|\cos \pi x| + C}$$

without table →

$$9.\ \frac{1}{2}\int \frac{2dx}{x^2\sqrt{4x^2+9}} = \frac{1}{2}\int \frac{\cancel{3}\sec^{\cancel{2}}\theta\,d\theta}{\frac{9}{4}\tan^2\theta \cdot \cancel{3\sec\theta}} = \frac{1}{\cancel{2}}\cdot\frac{\overset{2}{\cancel{4}}}{9}\int \frac{\sec\theta}{\tan^2\theta}d\theta = \frac{2}{9}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}d\theta$$

$2x = 3\tan\theta$

$2dx = 3\sec^2\theta\,d\theta$

$x = \frac{3}{2}\tan\theta$

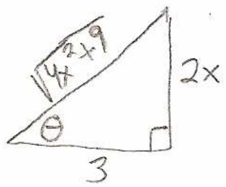
$$= \frac{2}{9}\int \frac{\cos\theta}{\sin^2\theta}d\theta = \frac{2}{9}\int u^{-2}du = \frac{2}{9}\cdot\frac{-1}{u} + C = -\frac{2}{9}\cdot\csc\theta + C$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= -\frac{\cancel{2}}{9}\cdot\frac{\sqrt{4x^2+9}}{\cancel{2}x} + C$$

$$= \boxed{-\frac{\sqrt{4x^2+9}}{9x} + C}$$



$\tan\theta = \frac{2x}{3}$

OR

with table entry 28 →

$$9.\ \frac{1}{2}\int \frac{2dx}{x^2\sqrt{4x^2+9}}$$

$\frac{u}{2} = x$

$u = 2x$

$a = 3$

$du = 2dx$

$$= \frac{1}{2}\int \frac{du}{\left(\frac{u^2}{4}\right)\sqrt{u^2+a^2}} = 2\int \frac{du}{u^2\sqrt{a^2+u^2}}$$

$$\boxed{28}\ \int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2 u} + C$$

$$\overset{\boxed{28}}{=} \cancel{2}\left[-\frac{\sqrt{9+4x^2}}{9(\cancel{2}x)}\right] = \boxed{-\frac{\sqrt{4x^2+9}}{9x} + C}$$

without table →

$$11.\ \int_{-1}^{0} t^2 e^{-t}\,dt = \left[-t^2 e^{-t}\right]_{-1}^{0} + 2\int t e^{-t}\,dt = (0+e) + 2\left[-te^{-t} + \int_{-1}^{0} e^{-t}\,dt\right]$$

$u = t^2$ $\quad v = -e^{-t}$

$du = 2t\,dt$ $\quad dv = e^{-t}dt$

$u = t$ $\quad v = -e^{-t}$

$du = dt$ $\quad dv = e^{-t}dt$

$$= e - 2\left[te^{-t}\right]_{-1}^{0} - 2\left[e^{-t}\right]_{-1}^{0}$$

$$= e - 2[0+e] - 2[1-e]$$

$$= e - \cancel{2e} - 2 + \cancel{2e}$$

$$= \boxed{e-2.}$$

7.6 homework continued

$\boxed{OR}$ with table entry 97 → 11. $\int_{-1}^{0} t^2 e^{-t}\,dt$ $u=t$, $n=2$, $a=-1$, $du=dt$

$$\overset{\boxed{97}}{=} \left[-t^2 e^{-t}\right]_{-1}^{0} - (-2)\int_{-1}^{0} t e^{-t}\,dt$$

$u=t$ $v=-e^{-t}$

$du=dt$ $dv=e^{-t}dt$

$$= [0+e] + 2\left[-te^{-t} + \int e^{-t}dt\right]$$

$$= e - 2\left[te^{-t}\right]_{-1}^{0} - 2\left[e^{-t}\right]_{-1}^{0}$$

$$= e - 2[0+e] - 2[1-e] = e - \cancel{2e} - 2 + \cancel{2e}$$

$$= \boxed{e-2}$$

$\boxed{97}$ $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

without table → 13. $-\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2}\,dz^{(-)} = -\int \tan^3 u\,du = -\int (\sec^2 u - 1)\tan u\,du$

$u=\frac{1}{z}$

$du=-\frac{1}{z^2}dz$

$$= -\int \tan u \sec^2 u\,du + \int \tan u\,du = -\frac{\tan^2 u}{2} + \int \tan u\,du = \boxed{-\frac{1}{2}\tan^2\left(\frac{1}{z}\right) - \ln\left|\cos\frac{1}{z}\right| + C}$$

$\boxed{OR}$ with table entry 69 → 13. $-\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2}\,dz^{(-)} = -\int \tan^3 u\,du \overset{\boxed{69}}{=} -\left[\frac{1}{2}\tan^2 u + \ln|\cos u|\right] = \boxed{-\frac{1}{2}\tan^2\left(\frac{1}{z}\right) - \ln\left|\cos\frac{1}{z}\right| + C}$

$u=\frac{1}{z}$, $du=-\frac{1}{z^2}dz$

without table → 15. $\int e^{2x}\arctan(e^x)\,dx = \int e^x \arctan(e^x)\cdot e^x\,dx = \int u \arctan u\,du$

$e^{2x} = e^x \cdot e^x$

$u=e^x$

$du=e^x dx$

$U=\arctan u$ $V=\frac{1}{2}u^2$

$dU=\frac{1}{1+u^2}du$ $dV=u\,du$

$$= \frac{1}{2}u^2\arctan u - \frac{1}{2}\int \frac{u^2}{1+u^2}\,du = \frac{1}{2}e^{2x}\arctan(e^x) - \frac{1}{2}\int\left[1-\frac{1}{u^2+1}\right]du$$

$$\begin{array}{r} 1-\frac{1}{u^2+1} \\ u^2+1\,\overline{)\,u^2\qquad\quad} \\ \underline{-(u^2+1)} \\ -1 \end{array}$$

$$= \frac{1}{2}e^{2x}\arctan(e^x) - \frac{1}{2}u + \frac{1}{2}\arctan u + C$$

$$= \boxed{\frac{1}{2}e^{2x}\tan^{-1}(e^x) - \frac{1}{2}e^x + \frac{1}{2}\tan^{-1}(e^x) + C}$$

$\boxed{\text{OR}}$

with table entry → 15. $\int e^{2x}\arctan(e^x)\,dx = \int e^{x}\cdot\arctan(e^x)\cdot e^x\,dx = \int u\arctan u\,du$

$u = e^x$

$du = e^x\,dx$

$$\underset{\uparrow\,\boxed{92}}{=} \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C = \boxed{\frac{e^{2x}+1}{2}\tan^{-1}(e^x) - \frac{e^x}{2} + C}$$

(same answer as without table)

without table (long) → 17. $\int y\sqrt{6+4y-4y^2}\,dy = \frac{1}{2}\int y\sqrt{7-(2y-1)^2}\,dy\,(2) = \frac{1}{2}\int\frac{u+1}{2}\sqrt{7-u^2}\,du = \frac{1}{4}\int(u+1)\sqrt{7-u^2}\,du$

$u = 2y-1$

$du = 2dy$

$\rightarrow \frac{u+1}{2} = y$

$-4y^2+4y+6$

$= -4(y^2-y \qquad)+6$

$= -4(y^2-y+\frac{1}{4}-\frac{1}{4})+6$

$= -4(y-\frac{1}{2})^2+1+6 = -2\cdot2\cdot(y-\frac{1}{2})(y-\frac{1}{2})+7 = -(2y-1)^2+7 = 7-(2y-1)^2$

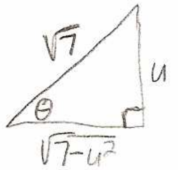
$$= \left(-\frac{1}{2}\right)\frac{1}{4}\int(-2)u\sqrt{7-u^2}\,du + \frac{1}{4}\int\sqrt{7-u^2}\,du$$

$x = 7-u^2$

$dx = -2u\,du$

$u = \sqrt{7}\sin\theta$

$du = \sqrt{7}\cos\theta\,d\theta$



$$= -\frac{1}{8}\int x^{\frac{1}{2}}\,dx + \frac{1}{4}\int\sqrt{7}\cos\theta\cdot\sqrt{7}\cos\theta\,d\theta = -\frac{1}{\underset{4}{\cancel{8}}}\cdot\frac{\cancel{2}}{3}x^{\frac{3}{2}} + \frac{7}{4}\cdot\frac{1}{2}\int[1+\cos2\theta]\,d\theta$$

$$= -\frac{1}{12}(7-u^2)^{\frac{3}{2}} + \frac{7}{8}\left[\theta + \frac{\overset{\cancel{2}\sin\theta\cos\theta}{\sin2\theta}}{\cancel{2}}\right] = -\frac{1}{12}(6+4y-4y^2)^{\frac{3}{2}} + \frac{7}{8}\sin^{-1}\left(\frac{u}{\sqrt{7}}\right) + \frac{7}{8}\cdot\frac{u}{\sqrt{7}}\cdot\frac{\sqrt{7-u^2}}{\sqrt{7}}$$

$$= \boxed{-\frac{1}{12}(6+4y-4y^2)^{\frac{3}{2}} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + \frac{1}{8}(2y-1)\sqrt{6+4y-4y^2} + C}$$

with table entry 30 near end (still very long) → 17. $\int y\sqrt{6+4y-4y^2}\,dy \underset{\uparrow\text{ from above work}}{=} \frac{1}{2}\int y\sqrt{7-(2y-1)^2}\,dy^{(2)} = \frac{1}{2}\int\frac{u+1}{2}\sqrt{7-u^2}\,du = \frac{1}{4}\int(u+1)\sqrt{7-u^2}\,du$

$u = 2y-1 \rightarrow \frac{u+1}{2} = y$

$du = 2dy$

$$= \left(-\frac{1}{2}\right)\frac{1}{4}\int u\sqrt{7-u^2}\,du^{(-2)} + \frac{1}{4}\underbrace{\int\sqrt{7-u^2}\,du}_{\text{using table entry }\boxed{30}\text{ with }a=\sqrt{7}} = -\frac{1}{8}\int x^{\frac{1}{2}}\,dx + \frac{1}{4}\left[\frac{u}{2}\sqrt{7-u^2} + \frac{7}{2}\sin^{-1}\frac{u}{\sqrt{7}}\right]$$

$x = 7-u^2$

$dx = -2u\,du$

$$\int\sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$= -\frac{1}{\underset{4}{\cancel{8}}}\frac{\cancel{2}}{3}x^{\frac{3}{2}} + \frac{1}{8}(2y-1)\sqrt{6+4y-4y^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) = \boxed{-\frac{1}{12}(6+4y-4y^2)^{\frac{3}{2}} + \frac{1}{8}(2y-1)\sqrt{6+4y-y^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + C}$$

7.6 homework continued

without table

$$19.\ \int \sin^2 x \cos x \ln(\sin x)\,dx = \int u^2 \ln u\,du = \frac{1}{3}u^3 \ln u - \int \frac{1}{3}u^2\,du = \frac{1}{3}\sin^3 x \ln(\sin x) - \frac{u^3}{9} + C$$

$u = \sin x,\ du = \cos x\,dx$

$U = \ln u \quad V = \frac{u^3}{3}$

$dU = \frac{1}{u}du \quad dV = u^2\,du$

$$= \boxed{\frac{1}{3}\sin^3 x \ln(\sin x) - \frac{1}{9}\sin^3 x + C}$$

OR

with table entry 101

$$19.\ \int \sin^2 x \cos x \ln(\sin x)\,dx = \int u^2 \ln u\,du \overset{\boxed{101}}{=} \frac{u^3}{9}\left[3\ln u - 1\right] + C = \boxed{\frac{\sin^3 x}{9}\left[3\ln(\sin x) - 1\right]}$$

$u = \sin x,\ du = \cos x\,dx \qquad n = 2$

$$\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right]$$

without table

$$21.\ \int \frac{e^x}{3 - e^{2x}}\,dx = \int \frac{du}{3 - u^2} = \int \frac{1}{(\sqrt{3}+u)(\sqrt{3}-u)}\,du = \int \frac{A}{\sqrt{3}+u} + \frac{B}{\sqrt{3}-u}\,du$$

$u = e^x,\ du = e^x\,dx$

$1 = A(\sqrt{3} - u) + B(\sqrt{3} + u)$

$u = \sqrt{3} \Rightarrow 1 = 2\sqrt{3}B \Rightarrow B = \frac{1}{2\sqrt{3}}$

$u = -\sqrt{3} \Rightarrow 1 = 2\sqrt{3}A \Rightarrow A = \frac{1}{2\sqrt{3}}$

$$= \frac{1}{2\sqrt{3}}\ln|u + \sqrt{3}| + \frac{(-)1}{2\sqrt{3}}\int \frac{1}{\sqrt{3}-u}(-)du$$

$y = \sqrt{3} - u$

$dy = -du$

$$= \frac{1}{2\sqrt{3}}\ln|u + \sqrt{3}| - \frac{1}{2\sqrt{3}}\int \frac{1}{y}\,dy$$

$$= \frac{1}{2\sqrt{3}}\ln|e^x + \sqrt{3}| - \frac{1}{2\sqrt{3}}\ln|\sqrt{3} - u| + C = \frac{1}{2\sqrt{3}}\ln|e^x + \sqrt{3}| - \frac{1}{2\sqrt{3}}\ln|\sqrt{3} - e^x| + C$$

$$= \boxed{\frac{1}{2\sqrt{3}}\ln\left|\frac{e^x + \sqrt{3}}{\sqrt{3} - e^x}\right| + C}$$

OR

with table entry 19

$$21.\ \int \frac{e^x}{3 - e^{2x}}\,dx = \int \frac{du}{3 - u^2} = \frac{1}{2\sqrt{3}}\ln\left|\frac{u + \sqrt{3}}{u - \sqrt{3}}\right| + C = \boxed{\frac{1}{2\sqrt{3}}\ln\left|\frac{e^x + \sqrt{3}}{e^x - \sqrt{3}}\right| + C}$$

$u = e^x,\ du = e^x\,dx \qquad a = \sqrt{3}$

$$\boxed{19}\ \int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u + a}{u - a}\right|$$

without table → 23. $\int \sec^5 x\,dx = \int \sec^3 x \cdot \sec^2 x\,dx = \tan x \sec^3 x - \int 3\tan^2 x \sec^3 x\,dx$

$u = \sec^3 x \qquad v = \tan x$

$du = 3\sec^2 x \cdot \sec x \tan x\,dx \qquad dv = \sec^2 x\,dx$

$= \tan x \sec^3 x - 3\int (\sec^2 x - 1)\sec^3 x\,dx = \tan x \sec^3 x - 3\int \sec^5 x\,dx + 3\int \sec^3 x\,dx$

$\therefore 4\int \sec^5 x\,dx = \tan x \sec^3 x\,dx + 3\int \sec^3 x\,dx \Rightarrow \int \sec^5 x\,dx = \frac{1}{4}\tan x \sec^3 x + \frac{3}{4}\cdot\frac{1}{2}(\sec x \tan x + \ln|\sec + \tan|)$

$= \boxed{\frac{1}{4}\tan x \sec x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C}$

OR

with table entry 77 → 23. $\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

so $\int \sec^5 x\,dx = \frac{1}{4}\tan x \sec^3 x + \frac{3}{4}\int \sec^3 x\,dx$

$= \frac{1}{4}\tan x \sec^3 x + \frac{3}{4}\cdot\frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$

$= \boxed{\frac{1}{4}\tan x \sec^3 x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C}$

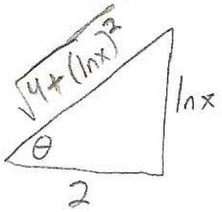
without table → 25. $\int \frac{\sqrt{4 + (\ln x)^2}}{x}\,dx = \int \frac{2\sec\theta}{} \cdot 2\sec^2\theta\,d\theta = 4\cdot\frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)$

$= \cancel{2}\cdot\frac{\sqrt{4 + (\ln x)^2}}{2}\cdot\frac{\ln x}{\cancel{2}} + 2\ln\left|\frac{\sqrt{4+(\ln x)^2}}{2} + \frac{\ln x}{2}\right|$

$\ln x = 2\tan\theta$

$\frac{1}{x}dx = 2\sec^2\theta\,d\theta$

$= \boxed{\frac{1}{2}\ln x\sqrt{4 + (\ln x)^2} + 2\ln\left|\sqrt{4+(\ln x)^2} + \ln x\right| + C}$



OR

with Table entry 21 → 25. $\int \frac{\sqrt{4 + (\ln x)^2}}{x}\,dx = \int \sqrt{4 + u^2}\,du \overset{\boxed{21}}{=} \frac{u}{2}\sqrt{4+u^2} + 2\ln\left(u + \sqrt{4+u^2}\right) + C$

$a = 2$

$u = \ln x$

$du = \frac{1}{x}dx$

Entry $\boxed{21}$: $\int \sqrt{a^2 + u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u + \sqrt{a^2+u^2}\right) + C$

$= \boxed{\frac{\ln x}{2}\sqrt{4 + (\ln x)^2} + 2\ln\left(\ln x + \sqrt{4 + (\ln x)^2}\right) + C}$

7.6 homework continued

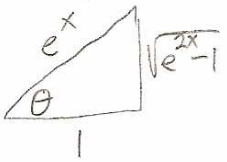
without table 27. $\int \sqrt{e^{2x}-1}\,dx = \int \sqrt{(e^x)^2-1}\,dx = \int \tan\theta \cdot \tan\theta\,d\theta = \int (\sec^2\theta-1)\,d\theta = \sec\theta\tan\theta - \theta + C$

$e^x = \sec\theta$

$e^x\,dx = \sec\theta\tan\theta\,d\theta$

$\cancel{\sec\theta}\,dx = \cancel{\sec\theta}\tan\theta\,d\theta$

$dx = \tan\theta\,d\theta$



$= \boxed{e^x\sqrt{e^{2x}-1} - \sec^{-1}(e^x) + C}$

$\boxed{\text{OR}}$

with table entry 41 27. $\int \sqrt{(e^x)^2-1}\,dx = \int \frac{\sqrt{u^2-1}}{u}\,du \overset{\substack{a=1 \\ \boxed{41}}}{=} \sqrt{u^2-1} - \cos^{-1}\frac{1}{|u|} + C$

$= \boxed{\sqrt{e^{2x}-1} - \cos^{-1}\frac{1}{|e^x|} + C}$

$u = e^x$

$du = e^x\,dx \rightarrow \frac{du}{e^x} = dx \rightarrow \frac{du}{u} = dx$

$u^2 = e^{2x}$

$= \sqrt{e^{2x}-1} - \cos^{-1}\left[(e^x)^{-1}\right] + C$

$\boxed{41}: \int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{|u|} + C$

$= \boxed{\sqrt{e^{2x}-1} + \cos^{-1}(e^x) + C}$

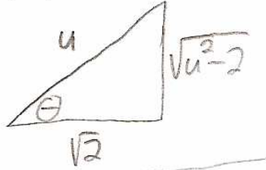
without table 29. $\int \frac{x^4\,dx}{\sqrt{x^{10}-2}} = \frac{1}{5}\int \frac{5x^4\,dx}{\sqrt{(x^5)^2-2}} = \frac{1}{5}\int \frac{du}{\sqrt{u^2-2}} = \frac{1}{5}\int \frac{\cancel{\sqrt{2}}\sec\theta\cancel{\tan\theta}\,d\theta}{\cancel{\sqrt{2}}\cancel{\tan\theta}} = \frac{1}{5}\ln|\sec\theta+\tan\theta|$

$u = x^5$

$du = 5x^4\,dx$

$u = \sqrt{2}\sec\theta$

$du = \sqrt{2}\sec\theta\tan\theta\,d\theta$



$= \frac{1}{5}\ln\left|\frac{u}{\sqrt{2}} + \frac{\sqrt{u^2-2}}{\sqrt{2}}\right| + C = \frac{1}{5}\ln\left|u+\sqrt{u^2-2}\right| + C = \boxed{\frac{1}{5}\ln\left|x^5+\sqrt{x^{10}-2}\right| + C}$

$\boxed{\text{OR}}$

with table entry 43 29. $\int \frac{x^4\,dx}{\sqrt{x^{10}-2}} = \frac{1}{5}\int \frac{5x^4\,dx}{\sqrt{(x^5)^2-2}} = \frac{1}{5}\int \frac{du}{\sqrt{u^2-2}} \overset{\boxed{43}}{=} \frac{1}{5}\ln\left|u+\sqrt{u^2-2}\right| + C$

$u = x^5$

$du = 5x^4\,dx$

$\boxed{43}: \int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$

$= \boxed{\frac{1}{5}\ln\left|x^5+\sqrt{x^{10}-2}\right| + C}$