7.5 evens

2. $\int \frac{\sin^3 x}{\cos x}dx = -\int \frac{1-\cos^2 x}{\cos x}(-\sin x\,dx) = -\int\left[\frac{1}{u}-u\right]du = \int\left[u-\frac{1}{u}\right]du = \frac{u^2}{2}-\ln|u|+C$

$= \boxed{\frac{1}{2}\cos^2 x - \ln|\cos x| + C}$

$u=\cos x,\ du=-\sin x\,dx$

4. $\int \tan^3\theta\,d\theta = \int(\sec^2\theta-1)\tan\theta\,d\theta = \int\tan\theta\sec^2\theta\,d\theta - \int\tan\theta\,d\theta = \boxed{\frac{1}{2}\tan^2\theta - \ln|\sec\theta|+C}$

or $\boxed{\frac{1}{2}\sec^2\theta+\ln|\cos\theta|+c}$

6. $\frac{1}{2}\int\frac{2x}{\sqrt{3-x^4}}dx = \frac{1}{2}\int\frac{\sqrt{3}\cos\theta\,d\theta}{\sqrt{3}\cos\theta} = \frac{1}{2}\theta+C = \boxed{\frac{1}{2}\sin^{-1}\left(\frac{x^2}{\sqrt{3}}\right)+C}$

$x^2=\sqrt{3}\sin\theta$

$2x\,dx=\sqrt{3}\cos\theta\,d\theta$

8. $\int x\csc x\cot x\,dx = -x\csc x+\int\csc x\,dx = \boxed{-x\csc x+\ln|\csc x-\cot x|+C}$

$u=x \qquad v=-\csc x$

$du=dx \qquad dv=\csc x\cot x\,dx$

10. $\int_0^4\frac{x-1}{x^2-4x-5}dx = \int\frac{A}{x-5}+\frac{B}{x+1}$

$(x-5)(x+1)$

$x-1=A(x+1)+B(x-5)$

$x-1=(A+B)x+(A-5B)$

$A+B=1$, $A-5B=-1$ → $-A-B=-1$, $A-5B=-1$, $-6B=-2$ → $\boxed{B=\frac{1}{3}}$ $\boxed{A=\frac{2}{3}}$

$= \left[\frac{2}{3}\ln|x-5|+\frac{1}{3}\ln|x+1|\right]_0^4 = \left(0+\frac{1}{3}\ln 5\right)-\left(\frac{2}{3}\ln 5+\frac{1}{\cancel{3}}0\right) = \boxed{-\frac{1}{3}\ln 5}$

12. $\frac{1}{2}\int\frac{2x}{x^4+x^2+1}dx = \frac{1}{2}\int\frac{1}{u^2+u+1}du = \frac{1}{2}\int\frac{1}{(u+\frac{1}{2})^2+\frac{3}{4}}du = \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{3}{4}}}\tan^{-1}\left(\frac{u+\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)+C$

$u^2+u+\frac{1}{4}-\frac{1}{4}+1$

$(u+\frac{1}{2})^2+\frac{3}{4}$

$= \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{(x^2+\frac{1}{2})}{\sqrt{3}}\cdot 2\right)+C = \boxed{\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C}$

$u=\cancel{x}^2$

$du=2x\,dx$

14. $\int\frac{x^3}{\sqrt{1+x^2}}dx = \frac{1}{2}\int\frac{x^2\cdot 2x\,dx}{\sqrt{1+x^2}} = \frac{1}{2}\int\frac{(u-1)\,du}{u^{\frac{1}{2}}} = \frac{1}{2}\int\left[u^{\frac{1}{2}}-u^{-\frac{1}{2}}\right]du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]$

$u=1+x^2 \rightarrow u-1=x^2$

$= \boxed{\frac{1}{3}(1+x^2)^{\frac{3}{2}}-(1+x^2)^{\frac{1}{2}}+C}$

$du=2x\,dx$

16. $\int_0^{\frac{\sqrt{2}}{2}}\frac{x^2}{\sqrt{1-x^2}}dx = \int_0^{\frac{\pi}{4}}\frac{\sin^2\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}}(1-\cos 2\theta)\,d\theta = \frac{1}{2}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$

~~$u=1-x^2$~~

$= \frac{1}{2}\left[\left(\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}\right)-(0-0)\right] = \boxed{\frac{\pi}{8}-\frac{1}{4}}$

~~$du=-2x\,dx$~~

$x=\sin\theta$

$dx=\cos\theta\,d\theta$

18. $\frac{1}{2}\int\frac{2e^{2t}}{1+e^{4t}}dt = \frac{1}{2}\int\frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1}x+C = \boxed{\frac{1}{2}\tan^{-1}(e^{2t})+C}$

$x=e^{2t}$

$dx=2e^{2t}dt$

20. $\int e^2 dx = \boxed{xe^2+C}$

22. $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}} dx = \frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}} dy = \frac{1}{2}\int u^{-\frac{1}{2}} du = \frac{1}{2}\cdot 2u^{\frac{1}{2}}+c = (1+y^2)^{\frac{1}{2}}+c$

$= \boxed{\sqrt{1+(\ln x)^2}+C}$

$y = \ln x$
$dy = \frac{1}{x}dx$

$u = 1+y^2$
$du = 2y\,dy$

24. $\int \ln(x^2-1)\,dx = x\ln(x^2-1) - \int \frac{2x^2}{x^2-1} dx = x\ln(x^2-1) - \int \left[2+\frac{2}{x^2-1}\right] dx$

$u = \ln(x^2-1)$ $\quad v = x$
$du = \frac{1}{x^2-1}\cdot 2x\,dx$ $\quad dv = dx$

$$\begin{array}{r|l} & 2+\frac{2}{x^2-1} \\ x^2-1 & 2x^2 \\ & -(2x^2-2) \\ \hline & 2 \end{array}$$

$x\ln(x^2-1) - 2x - \int\left(\frac{A}{x+1}+\frac{B}{x-1}\right)dx$

$2 = A(x-1)+B(x+1)$
$2 = (A+B)x+(-A+B)$

$A+B=0$
$-A+B=2$
$2B=2$ $\boxed{B=1}$ $\boxed{A=-1}$

$= \boxed{x\ln(x^2-1) - 2x + \ln|x+1| - \ln|x-1| + C}$

26. $\int \frac{3x^2-2}{x^3-2x-8} dx = \boxed{\ln|x^3-2x-8|+C}$

$u = x^3-2x-8$
$du = 3x^2-2\,dx$

28. $\frac{1}{a}\int \sin\sqrt{at}\,\overset{(a)}{dt} = \frac{1}{a}\int \sin\sqrt{u}\,du = \frac{1}{a}\int \sin x\cdot 2x\,dx = \frac{2}{a}\int x\sin x\,dx$

$u = at$
$du = a\,dt$

$x = \sqrt{u}$
$x^2 = u$
$2x\,dx = du$

Parts: $U = x$ $\quad V = -\cos x$
$dU = dx$ $\quad dV = \sin x\,dx$

$= \frac{2}{a}\left[-x\cos x - \int -\cos x\,dx\right] = -\frac{2}{a}x\cos x + \frac{2}{a}\int \cos x\,dx = -\frac{2}{a}x\cos x + \frac{2}{a}\sin x + C$

$= -\frac{2}{a}\sqrt{u}\cos\sqrt{u} + \frac{2}{a}\sin\sqrt{u} + C = \boxed{-\frac{2}{a}\sqrt{at}\cos\sqrt{at} + \frac{2}{a}\sin\sqrt{at} + C}$

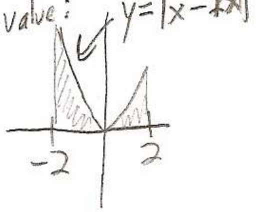
30. $\int_{-2}^{2} |x^2-4x|\,dx = \int_{-2}^{0}(x^2-4x)dx + \int_{0}^{2} -(x^2-4x)\,dx = \left[\frac{x^3}{3}-2x^2\right]_{-2}^{0} - \left[\frac{x^3}{3}-2x^2\right]_{0}^{2}$

$= (0-0) - \left(\frac{-8}{3}-8\right) - \left[\left(\frac{8}{3}-8\right)-(0-0)\right]$

$= \frac{8}{3}+8-\frac{8}{3}+8 = \boxed{16}$

$F(x) = x^2-4x$ $\quad \frac{4}{2}=2$
$x(x-4)$ $\quad (2,-4)$ vertex

Abs,value:



7.5 evens continued

32. $\frac{1}{2}\int \frac{\sqrt{2x-1}}{2x+3}\,dx\,(2) = \frac{1}{2}\int \frac{u}{u^2+4}\cdot 2u\,du = \int \frac{u^2}{u^2+4}\,du = \int\left[1 - \frac{4}{u^2+4}\right]du$

$u = \sqrt{2x-1}$

$u^2 = 2x-1 \rightarrow u^2+4 = 2x+3$

$2u\,du = 2\,dx$

$$u^2+4 \overline{)\,u^2} \quad 1 - \frac{4}{u^2+4}$$

$$\frac{-(u^2+4)}{-4}$$

$= u - 4\int \frac{1}{u^2+4}\,du$

$= u - 4\cdot\frac{1}{2}\tan^{-1}\frac{u}{2} + C$

$= u - 2\tan^{-1}\frac{u}{2} + C = \boxed{\sqrt{2x-1} - 2\tan^{-1}\left(\frac{\sqrt{2x-1}}{2}\right) + C}$

34. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+4\cot x}{4-\cot x}\,dx = \int \frac{\left[1+\frac{4\cos x}{\sin x}\right]\sin x}{\left[4-\frac{\cos x}{\sin x}\right]\sin x} = \int \frac{\sin x + 4\cos x}{4\sin x - \cos x} = \int \frac{1}{u}\,du = \Big[\ln|4\sin x - \cos x|\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$\cot x = \frac{\cos x}{\sin x}$

$u = 4\sin x - \cos x$

$du = 4\cos x + \sin x$

$= \ln|4-0| - \ln|4\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}|$

$= \boxed{\ln 4 - \ln\frac{3\sqrt{2}}{2}}$

36. $\int \sin 4x\cos 3x\,dx = \frac{1}{2}\int \sin x + \sin 7x\,dx = \frac{1}{2}\left[-\cos x - \frac{\cos 7x}{7}\right] + C = \boxed{-\frac{1}{2}\cos x - \frac{1}{14}\cos 7x + C}$

Product-sum ID: $\sin A\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$

38. $\int_0^{\frac{\pi}{4}} \tan^5\theta\sec^3\theta\,d\theta = \int_0^{\frac{\pi}{4}} \tan^4\theta\sec^2\theta\cdot\sec\theta\tan\theta\,d\theta = \int_0^{\frac{\pi}{4}} (\sec^2\theta - 1)^2\sec^2\theta\,\sec\theta\tan\theta\,d\theta$

$u = \sec\theta,\ du = \sec\theta\tan\theta\,d\theta$

$\cos\theta = \frac{1}{u}$

θ	u
0	1
$\frac{\pi}{4}$	$\sqrt{2}$

$= \int_1^{\sqrt{2}} (u^2-1)^2u^2\,du = \int_1^{\sqrt{2}} (u^4-2u^2+1)u^2\,du$

$= \int_1^{\sqrt{2}} (u^6 - 2u^4 + u^2)\,du = \left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}}$

$= \left(\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right) = \frac{120\sqrt{2} - 168\sqrt{2} + 70\sqrt{2} - 15 + 42 - 35}{105} = \frac{22\sqrt{2} - 8}{105}$

$= \boxed{\frac{2}{105}(11\sqrt{2} - 4)}$

40. $\int \frac{1}{\sqrt{4y^2-4y-3}}\,dy = \frac{1}{2}\int \frac{1\quad(2)\,dy}{\sqrt{(2y-1)^2-4}} = \frac{1}{2}\int \frac{\cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{2}\,\cancel{\tan\theta}}$

$2y-1 = 2\sec\theta$

$2dy = 2\sec\theta\tan\theta\,d\theta$

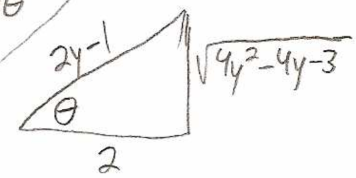
$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$4y^2-4y-3 = 4(y^2-y) - 3$

$= 4(y^2 - y + \frac{1}{4} - \frac{1}{4}) - 3$

$= 4(y-\frac{1}{2})^2 - 1 - 3 = 2(y-\frac{1}{2})\,2(y-\frac{1}{2}) - 4$

~~$= 4(y-\frac{1}{2})^2 - 4$~~ $= (2y-1)^2 - 4$



$= \frac{1}{2}\ln\left|\frac{2y-1}{2} + \frac{\sqrt{4y^2-4y-3}}{2}\right| + C$

$= \boxed{\frac{1}{2}\ln\left|2y-1 + \sqrt{4y^2-4y-3}\right| + C}$

42. $\int \frac{\tan^{-1}x}{x^2}dx = -\frac{1}{x}\tan^{-1}x + \int \frac{1}{x(1+x^2)}dx = -\frac{1}{x}\tan^{-1}x + \int\left[\frac{A}{x}+\frac{Bx+C}{x^2+1}\right]dx$

$1 = A(x^2+1)+(Bx+C)x$

$1 = (A+B)x^2+Cx+A$

$\boxed{A=1}$

$A+B=0$

$\boxed{B=-1}$ $\boxed{C=0}$

$u=\tan^{-1}x$ $v=\frac{-1}{x}$

$du=\frac{1}{1+x^2}dx$ $dv=x^{-2}dx$

$= -\frac{1}{x}\tan^{-1}x+\ln|x|-\frac{1}{2}\int\frac{2x}{x^2+1}dx$

$u=x^2+1$

$du=2x\,dx$

$= \boxed{-\frac{1}{x}\tan^{-1}x+\ln|x|-\frac{1}{2}\ln|x^2+1|+C.}$

44. $\int\sqrt{1+e^x}\,dx = \int\frac{\sqrt{1+e^x}\cdot e^x dx}{e^x} = \int\frac{u\cdot 2u\,du}{u^2-1} = 2\int\frac{u^2}{u^2-1}du$

$$\begin{array}{r} 1+\frac{1}{u^2-1} \\ u^2-1\,\overline{)\,u^2} \\ -(u^2-1) \\ \hline 1 \end{array}$$

~~$\sqrt{e^x}=e^{\frac{1}{2}x}=\tan\theta$~~

~~$\frac{1}{2}e^{\frac{1}{2}x}dx=\sec^2\theta\,d\theta$~~

$=2\int\left[1+\frac{1}{u^2-1}\right]du = 2u+2\int\left[\frac{A}{u+1}+\frac{B}{u-1}\right]du$

$1=A(u-1)+B(u+1)$

$u=1:\ 1=2B$ $\boxed{B=\frac{1}{2}}$

$u=-1:\ 1=-2A$ $\boxed{A=-\frac{1}{2}}$

$u=\sqrt{1+e^x}$

$u^2=1+e^x\to u^2-1=e^x$

$2u\,du=e^x dx$

$=2u+2\left[-\frac{1}{2}\ln|u+1|+\frac{1}{2}\ln|u-1|\right]$

$=\boxed{2\sqrt{1+e^x}-\ln\left|\sqrt{1+e^x}+1\right|+\ln\left|\sqrt{1+e^x}-1\right|+C}$

46. $\int\frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \int\frac{1+2\sin x+\sin^2x}{\cos^2x}dx = \int\sec^2x+\overset{(-)}{2}\int\frac{-\sin x}{\cos^2x}+\int\tan^2x\,dx$

$u=\cos x$

$du=-\sin x$

$=\tan x-2\int u^{-2}du+\int(\sec^2x-1)dx$

$=\tan x-2\left[\frac{u^{-1}}{-1}\right]+\tan x-x = 2\tan x+\frac{2}{\cos x}-x = \boxed{2\tan x+2\sec x-x+C}$

48. $\frac{1}{2}\int\frac{2x}{x^4-a^4}dx = \frac{1}{2}\int\frac{du}{u^2-a^4} = \frac{1}{2}\int\left[\frac{A}{u+a^2}+\frac{B}{u-a^2}\right]dx$

$1=A(u-a^2)+B(u+a^2)$

$1=(A+B)u+(-Aa^2+Ba^2)$

$(x^2+a^2)(x^2-a^2)$

$(x+a)(x-a)$

$=\frac{1}{2}\left(-\frac{1}{2a^2}\right)\ln|u+a^2|+\frac{1}{2}\left(\frac{1}{2a^2}\right)\ln|u-a^2|$

$(-A+B)a^2=1$

$-A+B=\frac{1}{a^2}$

$\underline{A+B=0}$

$2B=\frac{1}{a^2}\Rightarrow\boxed{B=\frac{1}{2a^2}}$

$\boxed{A=-\frac{1}{2a^2}}$

$u=x^2$

$du=2x\,dx$

$=-\frac{1}{4a^2}\ln|x^2+a^2|+\frac{1}{4a^2}\ln|x^2-a^2|+C$

$=\boxed{\frac{1}{4a^2}\ln\left|\frac{x^2-a^2}{x^2+a^2}\right|+C}$

7.5 evens continued

$$50.\ \frac{1}{4}\int \frac{1}{x^2\sqrt{4x+1}}(4)\,dx = \frac{1}{4}\int \frac{2\cancel{u}\,du}{\frac{(u^2-1)^2}{16}\cdot \cancel{u}} = 8\int \frac{1}{(u^2-1)^2}\,du = 8\int\left[\frac{A}{u+1}+\frac{B}{(u+1)^2}+\frac{C}{u-1}+\frac{D}{(u-1)^2}\right]du$$

$u=\sqrt{4x+1}$

$u^2=4x+1$

$2u\,du=4dx$

$\frac{u^2-1}{4}=x$

$$1= A(u+1)(u-1)^2+B(u-1)^2+C(u-1)(u+1)^2+D(u+1)^2 \Rightarrow$$

$u=1$ gives $1=4D \Rightarrow \boxed{D=\frac{1}{4}}$

$u=-1$ gives $1=4B \Rightarrow \boxed{B=\frac{1}{4}}$

$$1= A(u+1)(u^2-2u+1)+B(u^2-2u+1)+C(u-1)(u^2+2u+1)+D(u^2+2u+1)$$

$$1= A(u^3\underbrace{-2u^2+u+u^2-2u}_{-u^2\quad -u}+1)+Bu^2-2Bu+B+C(u^3+2u^2+u-u^2-2u-1)+Du^2+2Du+D$$

$$1=\cancel{Au^3}-\cancel{Au^2}-\cancel{Au}+A+\cancel{Bu^2}-\cancel{2Bu}+B+\cancel{Cu^3}+\cancel{Cu^2}-\cancel{Cu}-C+\cancel{Du^2}+\cancel{2Du}+D$$

$$1=(A+C)u^3+(-A+B+C+D)u^2+(-A-2B-C+2D)u+(A+B-C+D)$$

$$\begin{aligned} A \quad +C \quad &= 0\\ -A+\tfrac{1}{4}+C+\tfrac{1}{4} &= 0 \end{aligned} \Rightarrow \begin{aligned} A+C&=0\\ -A+C&=-\tfrac{1}{2}\\ \hline 2C&=-\tfrac{1}{2} \end{aligned} \Rightarrow \boxed{C=-\frac{1}{4}}\ \boxed{A=\frac{1}{4}}$$

$$\therefore\ 8\left[\frac{1}{4}\ln|u+1|+\frac{1}{4}\cdot\frac{(u+1)^{-1}}{-1}-\frac{1}{4}\ln|u-1|+\frac{1}{4}\frac{(u-1)^{-1}}{-1}\right]$$

$$=2\ln|u+1|-\frac{2}{u+1}-2\ln|u-1|-\frac{2}{u-1}+C$$

$$=2\ln\left|\frac{u+1}{u-1}\right|-\frac{2}{u+1}-\frac{2}{u-1}+C=\boxed{2\ln\left|\frac{\sqrt{4x+1}+1}{\sqrt{4x+1}-1}\right|-\frac{2}{\sqrt{4x+1}+1}-\frac{2}{\sqrt{4x+1}-1}+C}$$

$$52.\ \int\frac{dx}{x(x^4+1)}=\frac{1}{2}\int\frac{2x\,dx}{x^2(x^4+1)}=\frac{1}{2}\int\frac{du}{u(u^2+1)}=\frac{1}{2}\int\left[\frac{A}{u}+\frac{Bu+C}{u^2+1}\right]du$$

$1=A(u^2+1)+(Bu+C)u$

$1=Au^2+A+Bu^2+Cu$

$A+B=0\rightarrow\boxed{B=-1}$

$\boxed{C=0}$

$\boxed{A=1}$

$u=x^2$

$du=2x\,dx$

$$=\frac{1}{2}\cdot\ln|u|+\frac{1}{2}\cdot\int\frac{-u}{u^2+1}\,du$$

$y=u^2+1$

$dy=2u\,du$

$$=\frac{1}{2}\ln|u|-\frac{1}{2}\cdot\frac{1}{2}\int\frac{2u\,du}{u^2+1}=\frac{1}{2}\ln|u|-\frac{1}{4}\int\frac{1}{y}\,dy=\frac{1}{2}\ln(x^2)-\frac{1}{4}\ln(u^2+1)+C$$

$$=\boxed{\frac{1}{2}\ln(x^2)-\frac{1}{4}\ln(x^4+1)+C}$$

$$=\frac{1}{4}\left[2\ln(x^2)-\ln(x^4+1)\right]$$

$$=\frac{1}{4}\left\{\ln\left(\frac{x^4}{x^4+1}\right)\right\}+C$$

54. $\int (x+\sin x)^2\,dx = \int x^2 + 2x\sin x + \sin^2 x\,dx = \frac{x^3}{3} + 2\left[-x\cos x + \int \cos x\,dx\right] + \frac{1}{2}\int (1-\cos 2x)\,dx$

$u = x \quad v = -\cos x$

$du = dx \quad dv = \sin x\,dx$

$= \boxed{\frac{x^3}{3} - 2x\cos x + 2\sin x + \frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

56. $\int \frac{dx}{\sqrt{x} + x\sqrt{x}} = \int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2\cancel{u}\,du}{\cancel{u}(1+u^2)} = 2\tan^{-1}u + C = \boxed{2\tan^{-1}\sqrt{x} + C}$

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

58. $\frac{1}{2}\int \frac{x\ln x}{\sqrt{x^2-1}}^{(2)} dx = \frac{1}{\cancel{2}}\int \frac{\ln x}{\cancel{u}}\cdot \cancel{2}u\,du = \int \ln x\,du = \int \ln\sqrt{u^2+1}\,du$

Parts:

$U = \ln\sqrt{u^2+1} \qquad V = u$

$dU = \frac{1}{\sqrt{u^2+1}}\cdot\frac{1}{\cancel{2}}(u^2+1)^{-\frac{1}{2}}\cdot\cancel{2}u\,du \qquad dV = du$

$= \frac{u}{u^2+1}du$

$u = \sqrt{x^2-1}$

$u^2 = x^2 - 1 \rightarrow u^2+1 = x^2 \quad \sqrt{u^2+1} = x$

$2u\,du = 2x\,dx$

$$\begin{array}{r|l} & 1 - \frac{1}{u^2+1} \\ \hline u^2+1 & u^2 \\ & -(u^2+1) \\ \hline & -1 \end{array}$$

$= u\ln\sqrt{u^2+1} - \int \frac{u^2}{u^2+1}du$

$= u\ln\sqrt{u^2+1} - \int \left[1 - \frac{1}{u^2+1}\right]du = u\ln\sqrt{u^2+1} - u + \tan^{-1}u + C = \boxed{\sqrt{x^2-1}\ln x - \sqrt{x^2-1} + \tan^{-1}\sqrt{x^2-1} + C.}$

60. $\frac{1}{2}\int \frac{2dx}{x^2\sqrt{4x^2-1}} = \frac{1}{2}\int \frac{\sec\theta\cancel{\tan\theta}\,d\theta}{\frac{1}{4}\sec^2\theta\cdot\cancel{\tan\theta}} = 2\int \cos\theta\,d\theta = 2\sin\theta + C = \frac{\cancel{2}\sqrt{4x^2-1}}{\cancel{2}x} + C = \boxed{\frac{\sqrt{4x^2-1}}{x} + C.}$

~~$t = \sqrt{4x^2-1}$~~

~~$t^2 = 4x^2 - 1 \rightarrow 2t\,dt = 8x\,dx$~~

~~$\frac{t^2+1}{4} = x^2 \quad t\,dt = 4x\,dx$~~

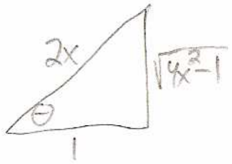
dead end.

$2x = \sec\theta$

$2dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{4x^2-1} = \tan\theta$

$x^2 = \frac{\sec^2\theta}{4}$



62. $\int \frac{1}{x + \sqrt[3]{x}}dx = \int \frac{1}{\sqrt[3]{x}\left(x^{\frac{2}{3}}+1\right)}dx = \int \frac{1}{u(u^2+1)}\cdot 3u^2\,du = \left(\frac{1}{2}\right)3\int \frac{2u}{u^2+1}du = \frac{3}{2}\ln(u^2+1) + C$

$\boxed{= \frac{3}{2}\ln\left(x^{\frac{2}{3}}+1\right) + C}$

$u = \sqrt[3]{x}$

$u^2 = x^{\frac{2}{3}}$

$du = \frac{1}{3}x^{-\frac{2}{3}}dx$

$du = \frac{1}{3x^{\frac{2}{3}}}dx$

$du = \frac{1}{3u^2}dx \Rightarrow 3u^2\,du = dx$

7.5 Evens continued

64. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(\tan x)}{\sin x \cos x}\,dx = \int_{0}^{\ln\sqrt{3}} u\,du = \left[\frac{1}{2}u^2\right]_0^{\ln\sqrt{3}} = \boxed{\frac{1}{2}(\ln\sqrt{3})^2}$ OR $= \frac{1}{2}\cdot\left[\frac{1}{2}\ln 3\right]^2 = \boxed{\frac{1}{8}(\ln 3)^2}$

$u = \ln(\tan x)$

$du = \frac{1}{\tan x}\cdot \sec^2 x\,dx = \frac{\cos x}{\sin x}\cdot\frac{1}{\cos^2 x}\,dx = \frac{1}{\sin x \cos x}\,dx.$

x	u
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	$\ln\sqrt{3}$

copied wrong!

copied wrong. 66. $\int_2^3 \frac{u^{\textcircled{2}}+1}{u^3-u^2}\,du = \int\left[\frac{A}{u}+\frac{B}{u^2}+\frac{C}{u-1}\right]$ $\quad u^2+1 = Au(u-1)+B(u-1)+Cu^2 = Au^2-Au+Bu-B+Cu^2$

$u^2+1 = (A+C)u^2+(-A+B)u-B$

$u^2(u-1)$

$A+C=1 \rightarrow -1+C=1 \rightarrow \boxed{C=2}$

$-A+B=0 \rightarrow \boxed{A=-1}$

$\boxed{B=-1}$

$= \left[-\ln|u| - \left(\frac{u^{-1}}{-1}\right) + 2\ln|u-1|\right]_2^3$

$= -\ln 3 + \frac{1}{3} + 2\ln 2 + \ln 2 - \frac{1}{2}$

$= \left(-\ln 3 + \frac{1}{3} + 2\ln 2\right) - \left(-\ln 2 + \frac{1}{2} + 0\right) = -\ln 3 + \frac{1}{3} + 2\ln 2 + \ln 2 - \frac{1}{2}$

$= 3\ln 2 - \ln 3 - \frac{1}{6} = \ln\left(\frac{8}{3}\right) - \frac{1}{6}.$

the real 66! 66. $\int_2^3 \frac{u^3+1}{u^3-u^2}\,du = \int_2^3 1\,du + \int_2^3 \frac{u^2+1}{u^3-u^2}\,du = \left[u\right]_2^3 + \ln\left(\frac{8}{3}\right) - \frac{1}{6} = 1 + \ln\frac{8}{3} - \frac{1}{6} = \boxed{\frac{5}{6} + \ln\left(\frac{8}{3}\right)}$

$$\begin{array}{r} 1 + \frac{u^2+1}{u^3-u^2} \\ u^3-u^2 \,\overline{\big)\, u^3+0u^2+0u+1} \\ \underline{-(u^3-u^2)} \\ u^2+1 \end{array}$$

68. $\int \frac{1}{1+2e^{x}-e^{-x}}\,dx = \int \frac{1}{1+2e^x-\frac{1}{e^x}}\,dx = \int \frac{e^x}{e^x+2e^{2x}-1}\,dx = \int \frac{e^x}{2e^{2x}+e^x-1}\,dx = \int \frac{e^x\,dx}{(2e^x-1)(e^x+1)}$

$u = e^x, \; du = e^x\,dx$

$= \int \frac{du}{(2u-1)(u+1)} = \int \frac{A}{2u-1} + \frac{B}{u+1}\,du \qquad 1 = A(u+1)+B(2u-1)$

$u=-1: \; 1 = -3B \Rightarrow \boxed{B=-\frac{1}{3}}$ OR $\begin{array}{l} A+2B=0 \\ A-B=1 \end{array} \rightarrow \begin{array}{r} -A-2B=0 \\ \underline{A-B=1} \\ -3B=1 \end{array} \boxed{B=-\frac{1}{3}}$

$u=\frac{1}{2}: \; 1 = \frac{3}{2}A \Rightarrow \boxed{A=\frac{2}{3}}$

$= \frac{2}{3}\left(\frac{1}{2}\right)\int\frac{(2)\,1}{2u-1}\,du - \frac{1}{3}\ln|u+1|$

$t = 2u-1, \; dt = 2du$

$= \frac{1}{3}\int\frac{1}{t}\,dt - \frac{1}{3}\ln|u+1| = \frac{1}{3}\ln|2u-1| - \frac{1}{3}\ln|u+1| = \boxed{\frac{1}{3}\ln|2e^x-1| - \frac{1}{3}\ln|e^x+1| + C}$ OR $\boxed{\frac{1}{3}\ln\left|\frac{2e^x-1}{e^x+1}\right| + C}$

70. $\int \frac{\ln(x+1)}{x^2}\,dx = -\frac{1}{x}\ln(x+1) + \int\frac{1}{x(x+1)}\,dx = -\frac{1}{x}\ln(x+1) + \int\left[\frac{A}{x}+\frac{B}{x+1}\right]dx$

$1 = Ax + A + Bx$

$\boxed{A=1}$

$A+B=0 \Rightarrow \boxed{B=-1}$

$= \boxed{-\frac{1}{x}\ln(x+1) + \ln|x| - \ln|x+1| + C.}$

Parts:

$u = \ln(x+1) \quad . \quad v = \frac{x^{-1}}{-1} = \frac{-1}{x}$

$du = \frac{1}{x+1}\,dx \quad dv = x^{-2}\,dx$

72. $\int \frac{4^x+10^x}{2^x}dx = \int [2^x+5^x]dx = \boxed{\frac{2^x}{\ln 2}+\frac{5^x}{\ln 5}+C}$

74. $2\int \frac{dx}{2\sqrt{x}(2+\sqrt{x})^4} = 2\int \frac{du}{u^4} = 2\int u^{-4}du = 2\left[\frac{u^{-3}}{-3}\right]+C = -\frac{2}{3u^3}+C = \boxed{-\frac{2}{(2+\sqrt{x})^3}+C}$

$u=2+\sqrt{x}$
$du=\frac{1}{2\sqrt{x}}dx$

76. $\int (x^2-bx)\sin 2x\,dx = -\frac{1}{2}\cos 2x(x^2-bx)+\frac{1}{2}\int(2x-b)\cos 2x\,dx$

$u=x^2-bx \quad V=\frac{-\cos 2x}{2}$
$du=(2x-b)dx \quad dv=\sin 2x\,dx$

$U=2x-b \quad V=\frac{\sin 2x}{2}$
$dU=2dx \quad dV=\cos 2x\,dx$

$= -\frac{1}{2}(x^2-bx)\cos 2x+\frac{1}{2}\left[\frac{(2x-b)\sin 2x}{2}-\int \sin 2x\,dx\right]$

$=\boxed{-\frac{1}{2}(x^2-bx)\cos 2x+\frac{1}{4}(2x-b)\sin 2x+\frac{\cos 2x}{4}+C}$

78. $\int \frac{\sec x\cos 2x}{\sin x+\sec x}dx = \int \frac{\left[\frac{1}{\cos x}\cdot\cos 2x\right]\cos x}{\left[\sin x+\frac{1}{\cos x}\right]\cos x}dx = \int \frac{\cos 2x}{\sin x\cos x+1} = \int \frac{2\cos 2x}{2\sin x\cos x+2}dx$

$=2\int \frac{\cos 2x}{\sin 2x+2}dx = \int \frac{1}{u}du = \boxed{\ln|\sin 2x+2|+C}$

$u=\sin 2x+2$
$du=2\cos 2x\,dx$

80. $\int \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx = \int \frac{\sin x\cos x\,dx}{(\sin^2 x)^2+(\cos^2 x)^2} = \frac{1}{2}\int \frac{2\sin x\cos x\,dx}{(\sin^2 x)^2+(1-\sin^2 x)^2} = \frac{1}{2}\int \frac{du}{u^2+(1-u)^2}$

$u=\sin^2 x$
$du=2\sin x\cos x\,dx$

$=\frac{1}{2}\int \frac{du}{u^2+1-2u+u^2} = \frac{1}{2}\int \frac{du}{2u^2-2u+1} = \int \frac{1}{(2u-1)^2+1}du$

$2(2u^2-2u+1)$
$=4(u^2-u+\frac{1}{4}-\frac{1}{4})+2$
$=4(u-\frac{1}{2})(u-\frac{1}{2})-1+2$
$=2(u-\frac{1}{2})2(u-\frac{1}{2})+1$
$=(2u-1)^2+1$