7.4 homework

1. a. $\dfrac{2x}{(x+3)(3x+1)} = \dfrac{A}{x+3} + \dfrac{B}{3x+1}$ b. $\dfrac{1}{x^3+2x^2+x} = \dfrac{1}{x(x^2+2x+1)} = \dfrac{1}{x(x+1)^2} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$.

3. a. $\dfrac{x^4+1}{x^5+4x^3} = \dfrac{x^4+1}{x^3(x^2+4)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{Dx+E}{x^2+4}$

b. $\dfrac{1}{(x^2-9)^2} = \dfrac{1}{[(x+3)(x-3)]^2} = \dfrac{1}{(x+3)^2(x-3)^2} = \dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2}$

5. a. $\dfrac{x^4}{x^4-1} = \dfrac{x^4}{(x^2+1)(x^2-1)} = \dfrac{x^4}{(x^2+1)(x+1)(x-1)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+1} + \dfrac{D}{x-1}$

b. $\dfrac{t^4+t^2+1}{(t^2+1)(t^2+4)^2} = \dfrac{Ax+B}{t^2+1} + \dfrac{Cx+D}{t^2+4} + \dfrac{Ex+F}{(t^2+4)^2}$

7. $\displaystyle\int \frac{x}{x-6}\,dx = \int \frac{u+6}{u}\,du = \int \left[1+\frac{6}{u}\right]du = u+6\ln|u|+C = x-6+6\ln|x-6|+C. = \boxed{x+6\ln|x-6|+C.}$

$u = x-6 \rightarrow u+6 = x$
$du = dx$

OR

$$\begin{array}{r} 1+\frac{6}{x-6} \\ x-6\,\overline{)\,x} \\ -(x-6) \\ \hline 6 \end{array}$$

$\displaystyle\int \frac{x}{x-6}\,dx = \int\left[1+\frac{6}{x-6}\right]dx = \boxed{x+6\ln|x-6|+C.}$

9. $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx = \int\left[\frac{2}{x+5} - \frac{1}{x-2}\right]dx = \boxed{2\ln|x+5| - \ln|x-2| + C.}$

$\dfrac{x-9}{(x+5)(x-2)} = \dfrac{A}{x+5} + \dfrac{B}{x-2} \Rightarrow x-9 = A(x-2)+B(x+5)$
$1x-9 = (A+B)x + (-2A+5B)$

$A+B=1$
$-2A+5B=-9$

$\xrightarrow{2R_1}$ $2A+2B=2$
$-2A+5B=-9$
$7B=-7$

$\boxed{B=-1}$
$A+(-1)=1$
$A=2.$

11. $\displaystyle\int_2^3 \frac{1}{x^2-1}\,dx = \int_2^3\left[\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\right]dx = \left[-\tfrac{1}{2}\ln|x+1| + \tfrac{1}{2}\ln|x-1|\right]_2^3 = \left(-\tfrac{1}{2}\ln 4 + \tfrac{1}{2}\ln 2\right) - \left(-\tfrac{1}{2}\ln 3 + 0\right)$

$= \boxed{-\tfrac{1}{2}\ln 4 + \tfrac{1}{2}\ln 2 + \tfrac{1}{2}\ln 3}$

$= \tfrac{1}{2}\left[\ln 3 + \ln 2 - \ln 4\right]$

$= \tfrac{1}{2}\ln\left[\tfrac{6}{4}\right]$

$= \boxed{\tfrac{1}{2}\ln\left(\tfrac{3}{2}\right)}$

$\dfrac{1}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1} \Rightarrow 1 = A(x-1)+B(x+1)$
$1 = (A+B)x + (-A+B)$

$A+B=0$
$-A+B=1$
$2B=1$

$\boxed{B=\tfrac{1}{2}} \Rightarrow A+\tfrac{1}{2}=0$
$\boxed{A=-\tfrac{1}{2}}$

13. $\displaystyle\int \frac{ax}{x^2-bx}\,dx = a\int \frac{x}{x(x-b)}\,dx = a\int \frac{1}{x-b}\,dx = \boxed{a\ln|x-b|+C.}$

15. $\displaystyle\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx = \int_3^4\left[1-\frac{4}{x^2(x-2)}\right]dx = \int_3^4 1\,dx - \int_3^4\left[\frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-2}\right]dx = \left[x\right]_3^4 + \left[\ln|x| + \frac{2x^{-1}}{-1} - \ln|x-2|\right]_3^4$

$= 1 + \left[\left(\ln 4 - \tfrac{1}{2} - \ln 2\right) - \left(\ln 3 - \tfrac{2}{3} - 0\right)\right]$

$= \tfrac{1}{2} + \ln 4 - \ln 2 - \ln 3 + \tfrac{2}{3} = \boxed{\tfrac{7}{6} + \ln\tfrac{2}{3}}$

$$\begin{array}{r} 1-\frac{4}{x^3-2x} \\ x^3-2x^2+0x+0\,\overline{)\,x^3-2x^2+0x-4} \\ -(x^3-2x^2) \\ \hline -4 \end{array}$$

$\dfrac{4}{x^2(x-2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2}$

$4 = Ax(x-2) + B(x-2) + Cx^2$
$4 = (A+C)x^2 + (-2A+B)x - 2B$

Equating coefficients: $-2B=4 \Rightarrow \boxed{B=-2} \Rightarrow -2A-2=0 \Rightarrow \boxed{A=-1} \Rightarrow -1+C=0 \Rightarrow \boxed{C=1}$

17. $\displaystyle\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \int_1^2 \left[\frac{2}{y} + \frac{\frac{9}{5}}{y+2} + \frac{\frac{1}{5}}{y-3}\right] dy = \left[2\ln|y| + \tfrac{9}{5}\ln|y+2| + \tfrac{1}{5}\ln|y-3|\right]_1^2$

$= \left[\left(2\ln 2 + \tfrac{9}{5}\ln 4 + 0\right) - \left(0 + \tfrac{9}{5}\ln 3 + \tfrac{1}{5}\ln 2\right)\right]$

$= 2\ln 2 - \tfrac{1}{5}\ln 2 + \tfrac{9}{5}\ln 4 - \tfrac{9}{5}\ln 3$

$= \boxed{\tfrac{9}{5}\ln 2 + \tfrac{9}{5}\ln 4 - \tfrac{9}{5}\ln 3} = \tfrac{9}{5}\ln 2 + \tfrac{9}{5}\ln(2^2) - \tfrac{9}{5}\ln 3$

$= \tfrac{9}{5}\ln 2 + \tfrac{18}{5}\ln 2 - \tfrac{9}{5}\ln 3 = \boxed{\tfrac{27}{5}\ln 2 - \tfrac{9}{5}\ln 3}$

$\dfrac{4y^2-7y-12}{y(y+2)(y-3)} = \dfrac{A}{y} + \dfrac{B}{y+2} + \dfrac{C}{y-3}$

$4y^2-7y-12 = A(y+2)(y-3) + By(y-3) + C_1(y+2)$

$y=-2 \Rightarrow 18 = 10B \Rightarrow \boxed{B = \tfrac{9}{5}}$

$y=3 \Rightarrow 3 = 15C \Rightarrow \boxed{C = \tfrac{1}{5}}$

$y=0 \Rightarrow -12 = -6A \Rightarrow \boxed{A=2}$

19. $\displaystyle\int \frac{1}{(x+5)^2(x-1)}\,dx = \int \left[\frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}\right] dx = -\frac{1}{36}\int \frac{1}{x+5}\,dx - \frac{1}{6}\int \frac{1}{(x+5)^2}\,dx + \frac{1}{36}\int \frac{1}{x-1}\,dx$

$= -\tfrac{1}{36}\ln|x+5| - \tfrac{1}{6}\cdot\dfrac{(x+5)^{-1}}{-1} + \tfrac{1}{36}\ln|x-1| + C$

$= \boxed{-\tfrac{1}{36}\ln|x+5| + \dfrac{1}{6(x+5)} + \tfrac{1}{36}\ln|x-1| + C.}$

$\dfrac{1}{(x+5)^2(x-1)} = \dfrac{A}{x+5} + \dfrac{B}{(x+5)^2} + \dfrac{C}{(x-1)}$

$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$

$x=-5 \Rightarrow 1 = -6B \Rightarrow \boxed{B = -\tfrac{1}{6}.}$

$x=1 \Rightarrow 1 = 36C \Rightarrow \boxed{C = \tfrac{1}{36}}$

$x=0 \Rightarrow 1 = -5A - B + 25C$

$1 = -5A + \tfrac{1}{6} + \tfrac{25}{36}$

$1 = -5A + \tfrac{31}{36}$

$\tfrac{5}{36} = -5A \Rightarrow A = \tfrac{5}{36}\cdot\left(-\tfrac{1}{5}\right)$

$\boxed{A = -\tfrac{1}{36}}$

OR (equating coefficients):

$1 = A(x^2+4x-5) + Bx - B + Cx^2 + 10Cx + 25C$

$1 = (A+C)x^2 + (4A+B+10C)x + (-5A-B+25C)$

$A + C = 0 \rightarrow \boxed{C=-A}$

$4A+B+10C = 0 \rightarrow 4A+B-10A=0$

$-5A-B+25C = 1 \rightarrow -5A-B-25A=1$

$-6A+B=0$

$-30A-B=1$

$-36A = 1 \rightarrow \boxed{A=-\tfrac{1}{36}}$

so, $\boxed{C=\tfrac{1}{36}}$

$-6\left(-\tfrac{1}{36}\right)+B=0$

$-\tfrac{1}{6}+B=0 \rightarrow \boxed{B=\tfrac{1}{6}}$

Need to know this integral:

$\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

21. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx = \int x\,dx + \int \frac{-4x+4}{x^2+4}\,dx = \frac{x^2}{2} - \tfrac{1}{2}\cdot 4\int \frac{2\cdot x}{x^2+4}\,dx + 4\int \frac{1}{x^2+4}\,dx$

$u = x^2+4,\ du = 2x\,dx$

$= \tfrac{1}{2}x^2 - 2\int \tfrac{1}{u}\,du + 4\cdot\tfrac{1}{2}\tan^{-1}\left(\tfrac{x}{2}\right) + C$

$= \tfrac{1}{2}x^2 - 2\ln|u| + 2\tan^{-1}\left(\tfrac{x}{2}\right) + C = \boxed{\tfrac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\tfrac{x}{2}\right) + C.}$

$$\begin{array}{r}
x + \frac{-4x+4}{x^2+4} \\
x^2+0x+4 \,\overline{)\, x^3+0x^2+0x+4} \\
-(x^3+0x^2+4x) \\
\hline
-4x+4
\end{array}$$

→ OR (equating coefficients): $4y^2-7y-12 = A(y^2-y-6) + By^2 - 3By + Cy^2 + 2Cy$

$= Ay^2 - Ay - 6A + By^2 - 3By + Cy^2 + 2Cy$

$= (A+B+C)y^2 + (-A-3B+2C)y + (-6A)$

$A+B+C = 4$

$-A-3B+2C = -7$

$-6A = -12 \rightarrow \boxed{A=2}$

$2+B+C=4$

$-2-3B+2C=-7$

$B+C=2$

$-3B+2C=-5$

$-2B-2C=-4$

$-3B+2C=-5$

$-5B = -9$

$\boxed{B=\tfrac{9}{5}}$

Using $B+C=2$,

$\tfrac{9}{5}+C=\tfrac{10}{5}$

$\boxed{C=\tfrac{1}{5}}.$

7.4 homework continued...

23. $\displaystyle\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx = \int\left[\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}\right]dx = 2\int\frac{1}{x}\,dx - \int\frac{1}{x^2}\,dx + 3\int\frac{1}{x+2}\,dx$

$x^3+2x^2 \to x^2(x+2)$

$= 2\ln|x| - \frac{x^{-1}}{-1} + 3\ln|x+2| + C$

$= \boxed{2\ln|x| + \frac{1}{x} + 3\ln|x+2| + C}$

$\displaystyle\frac{5x^2+3x-2}{x^2(x+2)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$

$5x^2+3x-2 = Ax(x+2)+B(x+2)+Cx^2 \longrightarrow$ OR $x=-2:\ 12=4C$, $\boxed{3=C}$

$= Ax^2+2Ax+Bx+2B+Cx^2$

$= (A+C)x^2+(2A+B)x+2B$

$x=0:\ -2=2B$, $\boxed{-1=B}$

$x=1:\ 6=3A-3+3$, $\boxed{A=2}$

$2B=-2$, $\boxed{B=-1}$

$2A+B=3$, $2A-1=3$, $2A=4$, $\boxed{A=2}$

$A+C=5$, $2+C=5$, $\boxed{C=3}$

25. $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx = \int\left[\frac{A}{x-1}+\frac{Bx+C}{x^2+9}\right]dx = \int\frac{1}{x-1}\,dx + \int\frac{-x-1}{x^2+9}\,dx = \ln|x-1| - \frac{1}{2}\int\frac{2x}{x^2+9}\,dx - \int\frac{dx}{x^2+9}$

$u=x^2+9$

$du=2x\,dx$

$10 = A(x^2+9)+(Bx+C)(x-1) \longrightarrow$ OR $x=1:\ 10=10A$, $\boxed{A=1}$

$10 = Ax^2+9A+Bx^2-Bx+Cx-C$

$10 = (A+B)x^2+(-B+C)x+(9A-C)$

$x=0:\ 10=9-C$, $\boxed{C=-1}$

$x=2:\ 10=13+(2B-1)\cdot 1$

$-3=2B-1$

$-2=2B \Rightarrow \boxed{B=-1}$

$A+B=0$

$-B+C=0$ $\Rightarrow A+C=0$

$9A-C=10$ $\to 9A-C=10$

$10A=10$

$\boxed{A=1} \Rightarrow \boxed{B=-1}$ and $\boxed{C=-1}$

$= \ln|x-1| - \frac{1}{2}\int\frac{1}{u}\,du - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

$= \boxed{\ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C}$

27. $\displaystyle\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}\,dx = \int\frac{Ax+B}{x^2+1}\,dx + \int\frac{Cx+D}{x^2+2}\,dx = \frac{1}{2}\int\frac{2x}{x^2+1}\,dx + \int\frac{1}{x^2+2}\,dx$

$u=x^2+1$

$du=2x\,dx$

$x^3+x^2+2x+1 = (Ax+B)(x^2+2)+(Cx+D)(x^2+1)$

$= Ax^3+Bx^2+2Ax+2B+Cx^3+Dx^2+Cx+D$

$= (A+C)x^3+(B+D)x^2+(2A+C)x+(2B+D)$

$= \frac{1}{2}\int\frac{1}{u}\,du + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$

$= \boxed{\frac{1}{2}\ln(x^2+1) + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C}$

$A + C = 1$

$B + D = 1$

$2A + C = 2$

$2B + D = 1$

$-A-C=-1$

$2A+C=2$

$\boxed{A=1} \Rightarrow \boxed{C=0}$

$-B-D=-1$

$2B+D=1$

$\boxed{B=0} \Rightarrow \boxed{D=1}$

u-substitution with some manipulations (see below for a simpler approach using completing the square)

29. $\int \frac{x+4}{x^2+2x+5}\,dx = \int \frac{x+1+3}{x^2+2x+5}\,dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+5}\,dx + 3\int \frac{1}{x^2+2x+5}\,dx$

$u = x^2+2x+5$
$du = 2x+2\,dx$
$du = 2(x+1)\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du + 3\int \frac{1}{(x+1)^2+4}\,dx$

$y = x+1$
$dy = dx$

Complete the square $x^2+2x+1-1+5$ $(x+1)^2+4$ to set it up for arctan.

$= \frac{1}{2}\ln|x^2+2x+5| + 3\int \frac{1}{y^2+4}\,dy = \frac{1}{2}\ln|x^2+2x+5| + 3\cdot\frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) + C$

$= \boxed{\frac{1}{2}\ln|x^2+2x+5| + \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C}$

(long way) 33. $\int_0^1 \frac{x^3+2x}{x^4+4x^2+3} = \int_0^1 \frac{Ax+B}{x^2+3}\,dx + \int_0^1 \frac{Cx+D}{x^2+1}\,dx = \frac{1}{2}\cdot\frac{1}{2}\int_0^1 \frac{2x}{x^2+3}\,dx + \frac{1}{2}\cdot\frac{1}{2}\int_0^1 \frac{2x}{x^2+1}\,dx$

$(x^2+3)(x^2+1)$

$u = x^2+3$
$du = 2x\,dx$

$y = x^2+1$
$dy = 2x\,dx$

$x^3+2x = (Ax+B)(x^2+1) + (Cx+D)(x^2+3)$
$= Ax^3+Bx^2+Ax+B+Cx^3+Dx^2+3Cx+3D$
$= (A+C)x^3+(B+D)x^2+(A+3C)x+(B+3D)$

$A+C=1 \rightarrow -A-C=-1$
$A+3C=2 \qquad A+3C=2$
$2C=1$
$\boxed{C=\frac{1}{2}}\ \boxed{A=\frac{1}{2}}$

$B+D=0 \rightarrow -B-D=0$
$B+3D=0 \qquad B+3D=0$
$2D=0$
$\boxed{D=0}\ \boxed{B=0}$

$= \frac{1}{4}\int_3^4 \frac{1}{u}\,du + \frac{1}{4}\int_1^2 \frac{1}{y}\,dy$

$= \frac{1}{4}\Big[\ln|u|\Big]_3^4 + \frac{1}{4}\Big[\ln|y|\Big]_1^2$

$= \frac{1}{4}(\ln 4 - \ln 3) + \frac{1}{4}(\ln 2)$

$= \boxed{\frac{1}{4}(\ln 4 - \ln 3 + \ln 2)} = \boxed{\frac{1}{4}\ln\left(\frac{8}{3}\right)}$

OR (short way) 33. $\frac{1}{4}\int_0^1 \frac{4(x^3+2x)}{x^4+4x^2+3}\,dx = \frac{1}{4}\int_3^8 \frac{1}{u}\,du = \frac{1}{4}\Big[\ln|u|\Big]_3^8$

$= \frac{1}{4}(\ln 8 - \ln 3) = \boxed{\frac{1}{4}\ln\left(\frac{8}{3}\right)}$

$u = x^4+4x^2+3$
$du = 4x^3+8x\,dx$
$du = 4(x^2+2x)\,dx$

OR ☆ 29. $\int \frac{x+4}{x^2+2x+5}\,dx = \int \frac{x+4}{(x+1)^2+4}\,dx = \int \frac{u-1+4}{u^2+4}\,du = \frac{1}{2}\int \frac{2\cdot u}{u^2+4}\,du + 3\int \frac{1}{u^2+4}\,du$

Most straightforward method.

comp. the □
$x^2+2x+1-1+5$

$u = x+1$
$du = dx$
$u-1 = x$

$= \frac{1}{2}\ln(u^2+4) + 3\cdot\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C$

$= \boxed{\frac{1}{2}\ln|x^2+2x+5| + \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C.}$

7.4 homework continued...

35. $\displaystyle\int \frac{dx}{x(x^2+4)^2} = \int\left[\frac{A}{x}+\frac{Bx+C}{x^2+4}+\frac{Dx+E}{(x^2+4)^2}\right]dx = \frac{1}{16}\int\frac{1}{x}\,dx - \frac{1}{2}\cdot\frac{1}{16}\int\frac{2x}{x^2+4}\,dx - \frac{1}{2}\cdot\frac{1}{4}\int\frac{2x\,dx}{(x^2+4)^2}$

$u = x^2+4$, $du = 2x\,dx$ $\qquad$ $y = x^2+4$, $dy = 2x\,dx$

$1 = A(x^2+4)^2 + (Bx+C)x(x^2+4) + (Dx+E)x$

$1 = A(x^4+8x^2+16) + (Bx^2+Cx)(x^2+4) + Dx^2+Ex$

$1 = Ax^4+8Ax^2+16A+Bx^4+Cx^3+4Bx^2+4Cx+Dx^2+Ex$

$1 = (A+B)x^4 + Cx^3 + (8A+4B+D)x^2 + (4C+E)x + 16A$

$16A=1$ $\quad \boxed{C=0}$ $\quad 8\cdot\frac{1}{16} - 4\cdot\frac{1}{16} + D = 0$ $\quad 4\cdot 0 + E = 0$

$\boxed{A=\frac{1}{16}}$ $\quad A+B=0$ $\quad \frac{1}{2}-\frac{1}{4}+D=0$ $\quad \boxed{E=0}$

$\boxed{B=-\frac{1}{16}}$ $\quad \frac{1}{4}+D=0$

$\boxed{D=-\frac{1}{4}}$

$= \frac{1}{16}\ln|x| - \frac{1}{32}\int\frac{1}{u}\,du - \frac{1}{8}\int y^{-2}\,dy$

$= \frac{1}{16}\ln|x| - \frac{1}{32}\ln(x^2+4) - \frac{1}{8}\frac{y^{-1}}{-1} + C$

$= \boxed{\frac{1}{16}\ln|x| - \frac{1}{32}\ln(x^2+4) + \frac{1}{8(x^2+4)} + C}$

Note: I could have canceled the us here to simplify.

39. $\displaystyle\int\frac{1}{x\sqrt{x+1}}\,dx = \int\frac{2u}{(u^2-1)\cdot u}\,du = \int\frac{2u}{u(u+1)(u-1)}\,du = \int\left[\frac{A}{u}+\frac{B}{u+1}+\frac{C}{u-1}\right]du$

$u=\sqrt{x+1}$ $\quad 2u = A(u+1)(u-1) + Bu(u-1) + Cu(u+1)$

$u^2 = x+1$ $\quad 2u = A(u^2-1) + Bu^2 - Bu + Cu^2 + Cu$

$u^2-1 = x$ $\quad 2u = Au^2 - A + Bu^2 - Bu + Cu^2 + Cu$

$2u\,du = dx$ $\quad 2u = (A+B+C)u^2 + (-B+C)u - A$

OR $u=1$: $2=2C \Rightarrow \boxed{C=1}$

$u=-1$: $-2=2B \Rightarrow \boxed{B=-1}$

$u=0$: $0=-A \Rightarrow \boxed{A=0}$

↑ faster on #39

$A+B+C=0$
$-B+C=2$
$-A=0 \to \boxed{A=0}$

$\begin{cases} B+C=0 \\ -B+C=2 \end{cases}$
$2C=2$
$\boxed{C=1}$ $\boxed{B=-1}$

$= -\int\frac{1}{u+1}\,du + \int\frac{1}{u-1}\,du$

$= -\ln|u+1| + \ln|u-1| + C$

$= -\ln\left(\sqrt{x+1}+1\right) + \ln\left|\sqrt{x+1}-1\right| + C$

$= \boxed{\ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C}$

long division after the substitution → 41. $\displaystyle\int_9^{16}\frac{\sqrt{x}}{x-4}\,dx = \int_3^4\frac{u\cdot 2u\,du}{u^2-4} = \int_3^4\frac{2u^2}{u^2-4}\,du = \int_3^4\left[2+\frac{8}{u^2-4}\right]du = \left[2u\right]_3^4 + \int_3^4\frac{8\,du}{(u+2)(u-2)}$

$u=\sqrt{x}$
$u^2=x$
$2u\,du=dx$

$$\begin{array}{r} 2+\frac{8}{u^2-4} \\ u^2-4\,\overline{)\,2u^2} \\ -(u^2-8) \\ \hline 8 \end{array}$$

$= (8-6) + \underbrace{\int_3^4\left[\frac{A}{u+2}+\frac{B}{u-2}\right]du}$

$8 = A(u-2) + B(u+2)$

$u=2$: $8=4B \Rightarrow \boxed{B=2}$

$u=-2$: $8=-4A \Rightarrow \boxed{A=-2}$

$= 2 - 2\int_3^4\frac{1}{u+2}\,du + 2\int_3^4\frac{1}{u-2}\,du.$

$= 2 - 2\left[\ln|u+2|\right]_3^4 + 2\left[\ln|u-2|\right]_3^4 = 2 - 2(\ln 6 - \ln 5) + 2(\ln 2) = \boxed{2 - 2\ln 6 + 2\ln 5 + 2\ln 2}$

$= 2 - \ln 6^2 + \ln 5^2 + \ln 2^2 = 2 - \ln 36 + \ln 25 + \ln 4 = 2 + \ln\left(\frac{100}{36}\right) = \boxed{2+\ln\left(\frac{25}{9}\right)}$