For #1 – #7, find the avg. value of the function on the given interval.

1. $f(x)=4x-x^2,\ [0,4]$. $\quad f_{avg}=\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-0}\int_0^4 [4x-x^2]\,dx = \frac{1}{4}\left[2x^2-\frac{x^3}{3}\right]_0^4 = \frac{1}{4}\left[32-\frac{64}{3}\right]$

$= \frac{1}{4}\left[\frac{96-64}{3}\right] = \frac{1}{4}\left[\frac{32}{3}\right] = \boxed{\frac{8}{3}}$

3. $g(x)=\sqrt[3]{x},\ [1,8]$. $\quad g_{avg}=\frac{1}{8-1}\int_1^8 x^{\frac{1}{3}}\,dx = \frac{1}{7}\cdot\frac{3}{4}\left[x^{\frac{4}{3}}\right]_1^8 = \frac{3}{28}\left[8^{\frac{4}{3}}-1\right] = \frac{3}{28}[16-1] = \frac{3}{28}\cdot 15 = \boxed{\frac{45}{28}}$

5. $f(t)=te^{-t^2},\ [0,5]$. $\quad f_{avg}=\left(-\frac{1}{2}\right)\frac{1}{5-0}\int_0^5 te^{-t^2}\,dt\,(-2) = -\frac{1}{10}\int_0^{-25} e^u\,du = -\frac{1}{10}\left[e^u\right]_0^{-25} = -\frac{1}{10}\left[e^{-25}-e^0\right]$

Let $u=-t^2$, $du=-2t\,dt$

t	u
0	0
5	−25

$= \boxed{-\frac{1}{10}\left[\frac{1}{e^{25}}-1\right]}$ OR $\boxed{\frac{1}{10}\left[1-\frac{1}{e^{25}}\right]}$

7. $h(x)=\cos^4 x\sin x,\ [0,\pi]$. $\quad h_{avg}=(-1)\frac{1}{\pi-0}\int_0^{\pi}\cos^4 x\sin x\,dx\,(-1) = -\frac{1}{\pi}\int_1^{-1} u^4\,du = -\frac{1}{\pi}\left[\frac{u^5}{5}\right]_1^{-1} =$ (next line)

$u=\cos x$, $du=-\sin x\,dx$

x	u
0	1
π	−1

$= -\frac{1}{\pi}\left[-\frac{1}{5}-\frac{1}{5}\right]$

$= -\frac{1}{\pi}\left[-\frac{2}{5}\right] = \boxed{\frac{2}{5\pi}}$

9. Given $f(x)=(x-3)^2$ on the interval $[2,5]$:

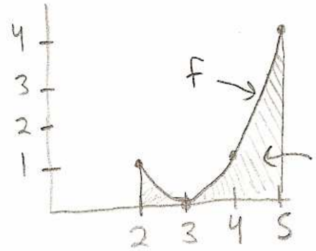
a) Find f_{avg}. $\quad f_{avg}=\frac{1}{5-2}\int_2^5 (x-3)^2\,dx = \frac{1}{3}\int_{-1}^{2} u^2\,du = \frac{1}{3}\left[\frac{u^3}{3}\right]_{-1}^{2} = \frac{1}{9}\left[2^3-(-1)^3\right] = \frac{1}{9}[8+1] = \boxed{1}$

$u=x-3,\ du=dx$

x	u
2	−1
5	2

b) Find c such that $f_{avg}=f(c)$. $\quad f_{avg}=f(c) \Rightarrow 1=(c-3)^2 \Rightarrow \pm 1 = c-3 \Rightarrow c=3\pm 1 \Rightarrow \boxed{c=4 \text{ or } 2.}$

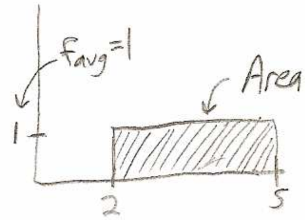
c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.



Area under f $= \int_2^5 (x-3)^2\,dx = \left.\frac{(x-3)^3}{3}\right]_2^5$

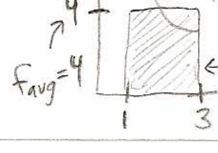
$= \frac{8}{3}-\left(-\frac{1}{3}\right) = 3.$

Rectangle whose area is the same as the area under f:



13. If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1,3]$.

$f_{avg}=\frac{1}{3-1}\int_1^3 f(x)\,dx = \frac{1}{2}(8) = 4$. Since $f_{avg}=4$, by the Mean Value Theorem for Integrals f must take on the value 4 on the interval $[1,3]$ at least once.



f must pass through y=4 somewhere between x=1 and x=3 for the area under f to equal the rectangle's area.

rectangle's area=8

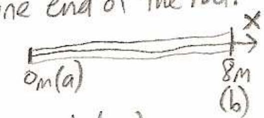
17. In a certain city the temperature (in °F) t hours after 9 a.m. was modeled by the function $T(t) = 50 + 14\sin\frac{\pi t}{12}$. Find the average temperature during the period from $\underbrace{9\text{am}}_{t=0}$ to $\underbrace{9\text{pm}}_{t=12}$.

$$T_{avg} = \frac{1}{12-0}\int_0^{12}\left[50 + 14\sin\frac{\pi t}{12}\right]dt = \frac{1}{12}\left[50t - \frac{14\cos\frac{\pi t}{12}}{\left(\frac{\pi}{12}\right)}\right]_0^{12} = \frac{1}{12}\left[50t - 14\cdot\frac{12}{\pi}\cos\frac{\pi t}{12}\right]_0^{12}$$

$$= \frac{1}{12}\left[\left(600 - \frac{168}{\pi}\underbrace{\cos\pi}_{-1}\right) - \left(0 - \frac{168}{\pi}\underbrace{\cos 0}_{1}\right)\right] = \frac{1}{12}\left[600 + \frac{168}{\pi} + \frac{168}{\pi}\right] = \frac{1}{12}\left[600 + \frac{336}{\pi}\right]$$

$$= \boxed{50 + \frac{28}{\pi}\,^{\circ}F} \approx \boxed{58.9^{\circ}F}$$

19. The linear density in a rod 8m long is $\frac{12}{\sqrt{x+1}}\ \frac{kg}{m}$, where x is measured in meters from one end of the rod.



Find the average density of the rod.

Using the avg. value formula, avg. density of the rod $= \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{8-0}\int_0^8 \frac{12}{\sqrt{x+1}}dx =$ (next line)

$$\frac{3}{2}\int_0^8 (x+1)^{-\frac{1}{2}}dx = \frac{3}{2}\int_1^9 u^{-\frac{1}{2}}du = \frac{3}{2}\left[2u^{\frac{1}{2}}\right]_1^9 = 3\left[3-1\right] = \boxed{6\ \frac{kg}{m}}$$

$u = x+1,\ du = dx$