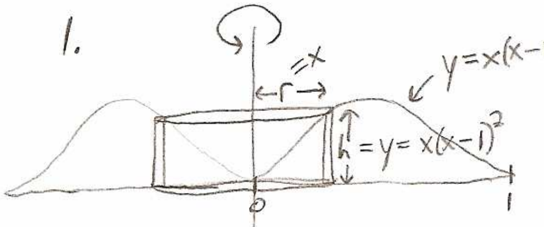
1.




$V=\int_{x_1}^{x_2} 2\pi r\cdot h\cdot dx = \int_0^1 2\pi x\cdot x(x-1)^2 dx = 2\pi\int_0^1 \left[x^2(x^2-2x+1)\right]dx$

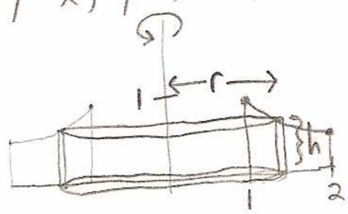
$= 2\pi\int_0^1 \left[x^4-2x^3+x^2\right]dx = 2\pi\left[\frac{x^5}{5}-\frac{x^4}{2}+\frac{x^3}{3}\right]_0^1 = 2\pi\left[\frac{1}{5}-\frac{1}{2}+\frac{1}{3}\right]$

$= 2\pi\left[\frac{6-15+10}{30}\right] = 2\pi\cdot\frac{1}{30} = \boxed{\frac{\pi}{15}}.$

3. $y=\frac{1}{x}$, $y=0$, $x=1$, $x=2$; about the y-axis.

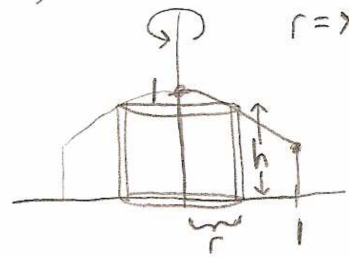
$r=x$, $h=y=\frac{1}{x}$. $V=\int_{x_1}^{x_2} 2\pi r\cdot h\cdot dx = \int_1^2 2\pi x\cdot\frac{1}{x}dx = 2\pi\int_1^2 dx = 2\pi\left[x\right]_1^2$

$= 2\pi\left[2-1\right] = \boxed{2\pi}$




5. $e^{-x^2}$, $y=0$, $x=0$, $x=1$; about the y-axis.

| x | y |
|---|---|
| 0 | $e^{-0}=e^0=1$ |
| 1 | $e^{-1}=\frac{1}{e}\approx .4$ |




$r=x$, $h=y=e^{-x^2}$. $V=\int_{x_1}^{x_2} 2\pi r\cdot h\cdot dx = -\int_0^1 -2\pi x e^{-x^2}dx$ $u=-x^2$, $du=-2x\,dx$

| x | u |
|---|---|
| 0 | 0 |
| 1 | -1 |

$= -\pi\int_0^{-1} e^u du = -\pi\left[e^u\right]_0^{-1}$

$= -\pi\left[e^{-1}-e^0\right] = -\pi\left[\frac{1}{e}-1\right]$

$= \boxed{\pi\left[1-\frac{1}{e}\right]}.$

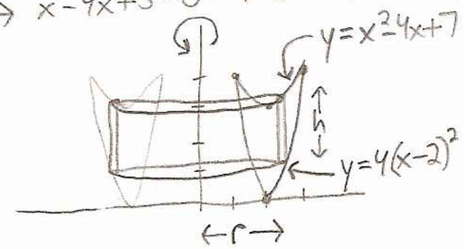
7. $y=4(x-2)^2$, $y=x^2-4x+7$; about the y-axis.

Curves intersect when $4(x-2)^2=x^2-4x+7 \rightarrow 4(x^2-4x+4)=x^2-4x+7 \rightarrow 4x^2-16x+16=x^2-4x+7 \rightarrow$

$3x^2-12x+9=0 \rightarrow x^2-4x+3=0 \rightarrow (x-3)(x-1)=0 \rightarrow x=1, x=3.$

| x | y |
|---|---|
| 1 | 4 |
| 2 | 0 |
| 3 | 4 |

| x | y |
|---|---|
| 1 | 4 |
| 2 | 3 |
| 3 | 4 |




$r=x$, $h=$ top function $-$ bottom function

$h=x^2-4x+7-4(x-2)^2$

$h=x^2-4x+7-4(x^2-4x+4)$

$h=x^2-4x+7-4x^2+16x-16$

$h=-3x^2+12x-9.$

$V=\int_{x_1}^{x_2} 2\pi r\cdot h\cdot dx = \int_1^3 2\pi x(-3x^2+12x-9)dx = 2\pi\int_1^3\left[-3x^3+12x^2-9x\right]dx = 2\pi\left[-\frac{3}{4}x^4+4x^3-\frac{9}{2}x^2\right]_1^3$

$= 2\pi\left[\left(-\frac{3}{4}\cdot 81+4\cdot 27-\frac{81}{2}\right)-\left(-\frac{3}{4}+4-\frac{9}{2}\right)\right] = 2\pi\left[\frac{-243}{4}+108-\frac{81}{2}+\frac{3}{4}-4+\frac{9}{2}\right]$

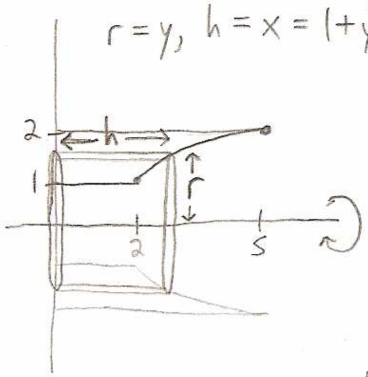
$= 2\pi\left[\frac{-240}{4}+104-\frac{72}{2}\right] = 2\pi\left[-60+104-36\right] = 2\pi\left[8\right] = \boxed{16\pi}.$

9. $x = 1+y^2,\ x=0,\ y=1,\ y=2$; about the x-axis.

$r = y,\ h = x = 1+y^2.$ $V = \int_{y_1}^{y_2} 2\pi r\cdot h\cdot dy = \int_1^2 2\pi y(1+y^2)\,dy = 2\pi\int_1^2 [y+y^3]\,dy$

$= 2\pi\left[\frac{y^2}{2}+\frac{y^4}{4}\right]_1^2 = 2\pi\left[(2+4)-\left(\frac{1}{2}+\frac{1}{4}\right)\right]$

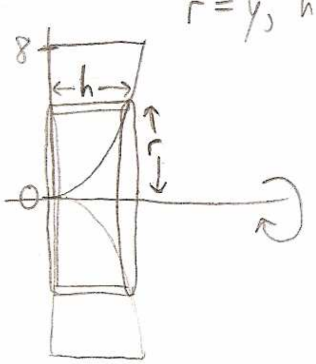
$= 2\pi\left[6-\frac{3}{4}\right] = 2\pi\left[\frac{24-3}{4}\right] = 2\pi\left[\frac{21}{4}\right] = \boxed{\frac{21\pi}{2}}$




11. $y=x^3,\ y=8,\ x=0$; about the x-axis.

$r=y,\ h=x=\sqrt[3]{y}.$ $V = \int_{y_1}^{y_2} 2\pi r\cdot h\cdot dy = \int_0^8 2\pi y\cdot\sqrt[3]{y}\,dy = 2\pi\int_0^8 y^{\frac{4}{3}}\,dy$

$= 2\pi\left[\frac{3}{7}y^{\frac{7}{3}}\right]_0^8 = \frac{6\pi}{7}\left[8^{\frac{7}{3}}-0\right] = \frac{6\pi}{7}[128] = \boxed{\frac{768\pi}{7}}$




13. $x = 1+(y-2)^2,\ x=2$; about the x-axis.

| x | y |
|---|---|
| 1 | 2 |
| 2 | 3 |
| 2 | 1 |

$r=y,\ h=$ right func. − left func. $= x_2-x_1 = 2-[(1+(y-2)^2]$

$h = 2-[1+y^2-4y+4]$

$h = 2-[y^2-4y+5]$

$h = 2-y^2+4y-5$

$h = -y^2+4y-3.$

$V = \int_{y_1}^{y_2} 2\pi r\cdot h\cdot dy$

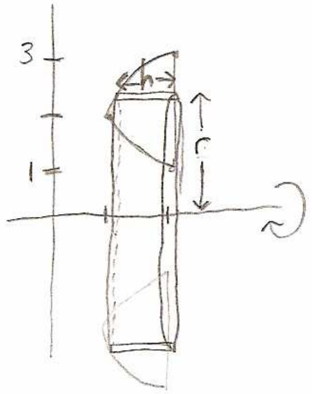
$V = \int_1^3 2\pi y(-y^2+4y-3)\,dy$

$V = 2\pi\int_1^3[-y^3+4y^2-3y]\,dy$

$V = 2\pi\left[-\frac{y^4}{4}+\frac{4}{3}y^3-\frac{3}{2}y^2\right]_1^3 = 2\pi\left[\left(-\frac{81}{4}+36-\frac{27}{2}\right)-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right]$

$= 2\pi\left[-\frac{81}{4}+36-\frac{27}{2}+\frac{1}{4}-\frac{4}{3}+\frac{3}{2}\right] = 2\pi\left[-\frac{80}{4}+36-\frac{24}{2}-\frac{4}{3}\right] = 2\pi\left[-20+36-12-\frac{4}{3}\right]$

$= 2\pi\left[4-\frac{4}{3}\right] = 2\pi\left[\frac{8}{3}\right] = \boxed{\frac{16\pi}{3}}.$

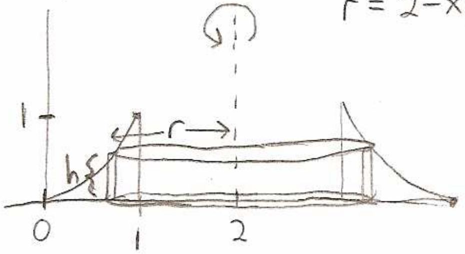



15. $y=x^4,\ y=0,\ x=1$; about $x=2$.

$r=2-x,\ h=y=x^4.$ $V = \int_{x_1}^{x_2} 2\pi r\cdot h\cdot dx = \int_0^1 2\pi(2-x)x^4\,dx$

$= 2\pi\int_0^1[2x^4-x^5]\,dx = 2\pi\left[\frac{2}{5}x^5-\frac{x^6}{6}\right]_0^1$

$= 2\pi\left[\frac{2}{5}-\frac{1}{6}\right] = \cancel{2}\pi\left[\frac{12-5}{\cancel{30}_{15}}\right] = \boxed{\frac{7\pi}{15}}$

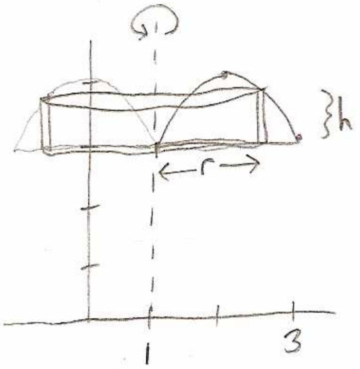


17. $y = 4x - x^2$, $y = 3$; about $x = 1$.

Curves intersect when $4x - x^2 = 3 \rightarrow 0 = x^2 - 4x + 3 \rightarrow 0 = (x-3)(x-1) \rightarrow x = 1, x = 3.$

| x | y |
|---|---|
| 1 | 3 |
| 2 | 4 |
| 3 | 3 |

$r = x - 1$, $h = y - 3 = 4x - x^2 - 3$.




$$V = \int_{x_1}^{x_2} 2\pi r \cdot h \cdot dx = \int_1^3 2\pi (x-1)(4x - x^2 - 3)\, dx$$

$$= 2\pi \int_1^3 [4x^2 - x^3 - 3x - 4x + x^2 + 3] dx = 2\pi \int_1^3 [-x^3 + 5x^2 - 7x + 3] dx$$

$$= 2\pi \left[ -\frac{1}{4}x^4 + \frac{5}{3}x^3 - \frac{7}{2}x^2 + 3x \right]_1^3$$

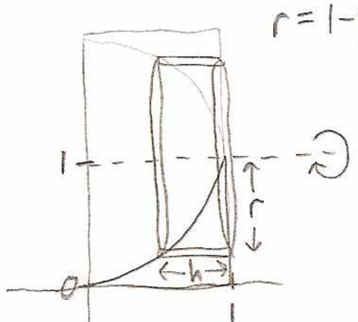
$$= 2\pi \left[ \left(-\frac{81}{4} + 45 - \frac{63}{2} + 9\right) - \left(-\frac{1}{4} + \frac{5}{3} - \frac{7}{2} + 3\right) \right]$$

$$= 2\pi \left[ -\frac{81}{4} + 54 - \frac{63}{2} + \frac{1}{4} - \frac{5}{3} + \frac{7}{2} - 3 \right] = 2\pi \left[ -\frac{80}{4} + 51 - \frac{56}{2} - \frac{5}{3} \right] = 2\pi \left[ -20 + 51 - 28 - \frac{5}{3} \right]$$

$$= 2\pi \left[ 3 - \frac{5}{3} \right] = 2\pi \left[ \frac{9-5}{3} \right] = 2\pi \left[ \frac{4}{3} \right] = \boxed{\frac{8\pi}{3}}$$

19. $y = x^3$, $y = 0$, $x = 1$; about $y = 1$.

$r = 1 - y$, $h = 1 - x = 1 - \sqrt[3]{y}$. $\quad V = \int_{y_1}^{y_2} 2\pi r \cdot h \cdot dy = \int_0^1 2\pi (1-y)(1 - \sqrt[3]{y})\, dy$

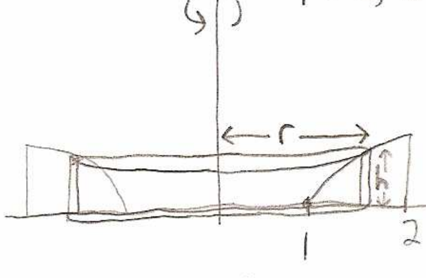



$$V = 2\pi \int_0^1 \left[1 - y^{\frac{1}{3}} - y + y^{\frac{4}{3}}\right] dy = 2\pi \left[ y - \frac{3}{4}y^{\frac{4}{3}} - \frac{y^2}{2} + \frac{3}{7}y^{\frac{7}{3}} \right]_0^1$$

$$= 2\pi \left[ 1 - \frac{3}{4} - \frac{1}{2} + \frac{3}{7} \right] = \cancel{2}\pi \left[ \frac{28 - 21 - 14 + 12}{\cancel{28}\ 14} \right] = \boxed{\frac{5\pi}{14}}$$

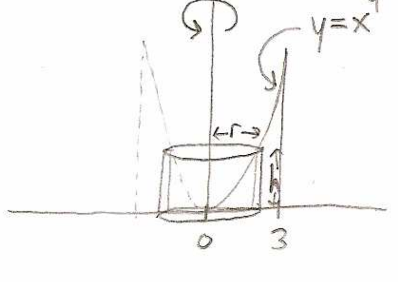
21. $y = \ln x$, $y = 0$, $x = 2$; about the $y$-axis.

$r = x$, $h = y = \ln x$. $\quad V = \int_{x_1}^{x_2} 2\pi r \cdot h \cdot dx = \boxed{\int_1^2 2\pi x \ln x\, dx}$



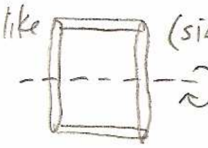

29. Describe the solid whose volume is represented by this integral: $\int_0^3 2\pi x^5\, dx$.

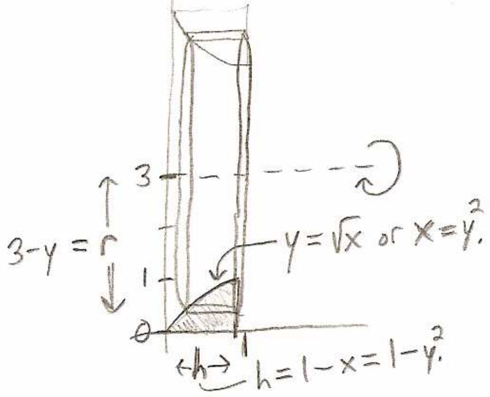
$$\int_0^3 2\pi x^5 dx = \int_0^3 2\pi \cdot x \cdot x^4 dx = \int_{x_1}^{x_2} 2\pi r \cdot h \cdot dx, \text{ so } r = x \text{ and } h = x^4.$$




This integral represents the volume of the solid obtained by rotating the region bounded by $y = x^4$, $y = 0$, $x = 0$, and $x = 0$ about the $y$-axis.

31. Describe the solid whose volume is represented by this integral: $\int_0^1 2\pi(3-y)(1-y^2)\,dy$

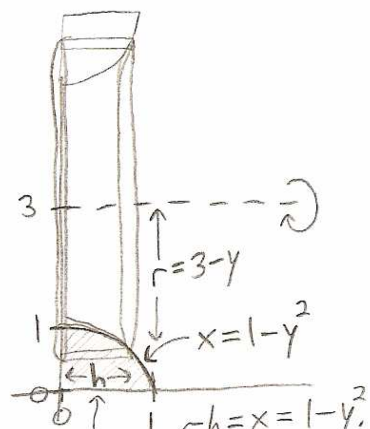
- One possibility: $\int_0^1 2\pi\underbrace{(3-y)}_{r}\underbrace{(1-y^2)}_{h}\,dy$ ← Since it is a $dy$ integral, the cylinder looks like  (sideways) and the rotation must be about a horizontal line.




This integral represents the volume of the solid obtained by rotating the region bounded by $y=\sqrt{x}$ (or $x=y^2$), $y=0$, and $x=1$ about the line $y=3$.

- Another possibility with $r=3-y$, $h=1-y^2$:

The integral also represents the volume of the solid obtained by rotating the region bounded by $x=1-y^2$, $x=0$, and $y=0$ about the line $y=3$.

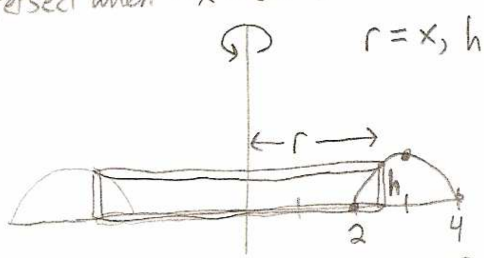



use cylindrical shell method 37. $y=-x^2+6x-8$, $y=0$; about the y-axis.

Curves intersect when $-x^2+6x-8=0 \rightarrow x^2-6x+8=0 \rightarrow (x-2)(x-4)=0 \rightarrow x=2, x=4.$

$r=x,\ h=y=-x^2+6x-8.$

| x | y |
|---|---|
| 2 | 0 |
| 3 | 1 |
| 4 | 0 |




$$V=\int_{x_1}^{x_2}2\pi r\cdot h\cdot dx=\int_2^4 2\pi x(-x^2+6x-8)\,dx$$

$$=2\pi\int_2^4\left[-x^3+6x^2-8x\right]dx=2\pi\left[\frac{-x^4}{4}+2x^3-4x^2\right]_2^4$$

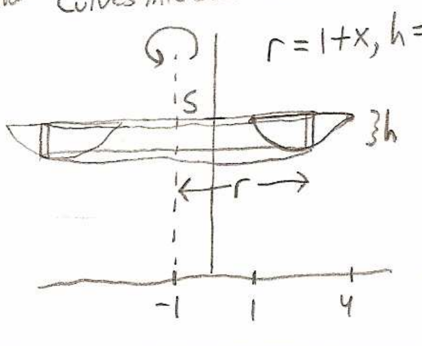
$$=2\pi\left[(-64+128-64)-(-4+16-16)\right]=2\pi[4]=\boxed{8\pi}.$$

use cylindrical shell method 39. $y=5$, $y=x+\frac{4}{x}$; about $x=-1$.

Curves intersect when $x+\frac{4}{x}=5 \rightarrow x^2+4=5x \rightarrow x^2-5x+4=0 \rightarrow (x-4)(x-1)=0 \rightarrow x=1, x=4.$

| x | y |
|---|---|
| 1 | 5 |
| 2 | 4 |
| 3 | $4\frac{1}{3}$ |
| 4 | 5 |

$r=1+x,\ h=5-y=5-x-\frac{4}{x}.$ $\quad V=\int_{x_1}^{x_2}2\pi r\cdot h\cdot dx=\int_1^4 2\pi(1+x)\left(5-x-\frac{4}{x}\right)dx$




$$=2\pi\int_1^4\left[5-x-\frac{4}{x}+5x-x^2-4\right]dx=2\pi\int_1^4\left[1+4x-\frac{4}{x}-x^2\right]dx$$

$$=2\pi\left[x+2x^2-4\ln|x|-\frac{x^3}{3}\right]_1^4=2\pi\left[\left(4+32-4\ln4-\frac{64}{3}\right)-\left(1+2-0-\frac{1}{3}\right)\right]$$

$$=2\pi\left[36-4\ln4-\frac{64}{3}-3+\frac{1}{3}\right]=2\pi\left[33-\underset{\substack{\uparrow\\21}}{\frac{63}{3}}-4\ln4\right]=\boxed{2\pi\left[12-4\ln4\right]}$$