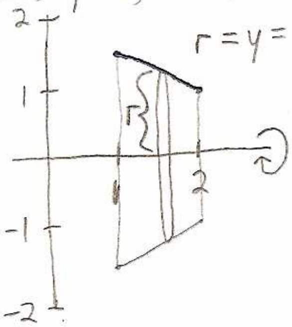
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

Disk method 1. $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, $x = 2$; about the x-axis.

x	y
1	1.5
2	1



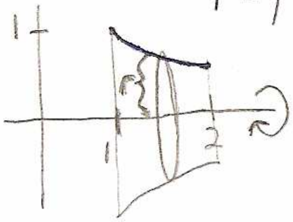
$r = y = 2 - \frac{1}{2}x$. $A(x) = \pi r^2 = \pi\left(2 - \frac{1}{2}x\right)^2 = \pi\left(4 - 2x + \frac{1}{4}x^2\right)$.

$$V = \int_{x_1}^{x_2} A(x)\,dx = \int_1^2 \pi\left[4 - 2x + \frac{1}{4}x^2\right]dx = \pi\left[4x - x^2 + \frac{1}{12}x^3\right]_1^2$$

$$= \pi\left[\left(8 - 4 + \frac{2}{3}\right) - \left(4 - 1 + \frac{1}{12}\right)\right] = \pi\left[4 + \frac{2}{3} - 3 - \frac{1}{12}\right] = \pi\left[1 + \frac{2}{3} - \frac{1}{12}\right]$$

$$= \pi\left[\frac{12 + 8 - 1}{12}\right] = \boxed{\frac{19\pi}{12}}$$

Disk method 3. $y = \frac{1}{x}$, $x = 1$, $x = 2$, $y = 0$; about the x-axis.

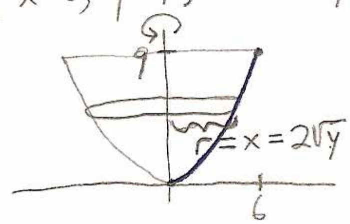


$r = y = \frac{1}{x}$. $A(x) = \pi r^2 = \pi\left(\frac{1}{x}\right)^2 = \pi x^{-2}$.

$$V = \int_{x_1}^{x_2} A(x)\,dx = \int_1^2 \pi x^{-2}\,dx = \pi\left[\frac{x^{-1}}{-1}\right]_1^2 = -\pi\left[\frac{1}{x}\right]_1^2 = -\pi\left[\frac{1}{2} - 1\right] = \boxed{\frac{\pi}{2}}$$

Disk method 5. $x = 2\sqrt{y}$, $x = 0$, $y = 9$; about the y-axis.

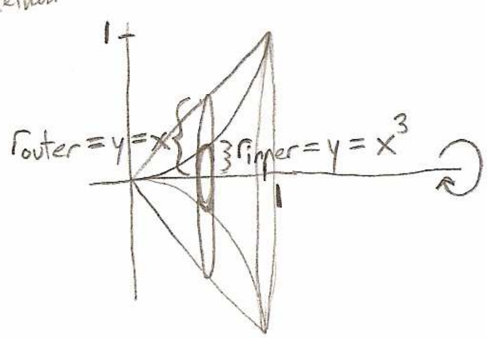
x	y
0	0
6	9



$A(y) = \pi r^2 = \pi(2\sqrt{y})^2 = \pi[4y]$.

$$V = \int_{y_1}^{y_2} A(y)\,dy = \int_0^9 \pi[4y]\,dy = \pi\left[2y^2\right]_0^9 = \pi[2 \cdot 81] = \boxed{162\pi}$$

Washer method 7. $y = x^3$, $y = x$, $x \geq 0$; about the x-axis.

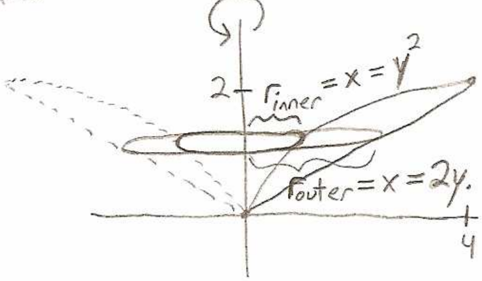


$$A(x) = \pi r_{outer}^2 - \pi r_{inner}^2 = \pi\left[x^2 - (x^3)^2\right] = \pi\left[x^2 - x^6\right].$$

$$V = \int_{x_1}^{x_2} A(x)\,dx = \int_0^1 \pi\left[x^2 - x^6\right]dx = \pi\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1 = \pi\left[\frac{1}{3} - \frac{1}{7}\right]$$

$$= \pi\left[\frac{7 - 3}{21}\right] = \boxed{\frac{4\pi}{21}}$$

Washer method 9. $y^2=x$, $x=2y$; about the y-axis. The curves intersect when $y^2=2y \Rightarrow y^2-2y=0 \Rightarrow y(y-2)=0$
$\Rightarrow y=0, y=2.$



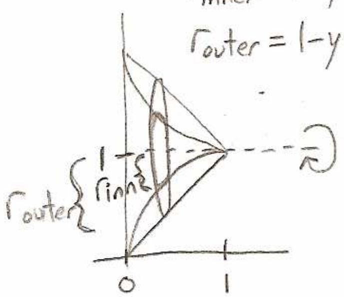
$$A(y)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[(2y)^2-(y^2)^2\right]=\pi\left[4y^2-y^4\right].$$

$$V=\int_{y_1}^{y_2}A(y)\,dy=\int_0^2\pi\left[4y^2-y^4\right]dy=\pi\left[\frac{4}{3}y^3-\frac{y^5}{5}\right]_0^2=\pi\left[\frac{32}{3}-\frac{32}{5}\right]$$

$$=\pi\left[\frac{160-96}{15}\right]=\boxed{\frac{64\pi}{15}}$$

Washer 11. $y=x$, $y=\sqrt{x}$; about $y=1$.

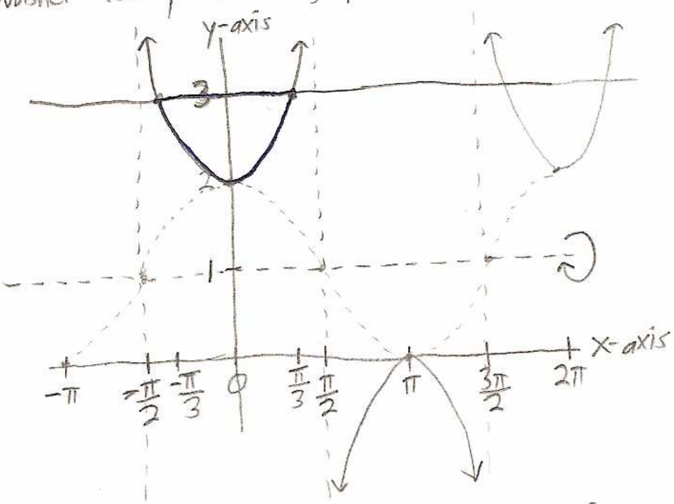
$r_{inner}=1-y=1-\sqrt{x}.$
$r_{outer}=1-y=1-x.$



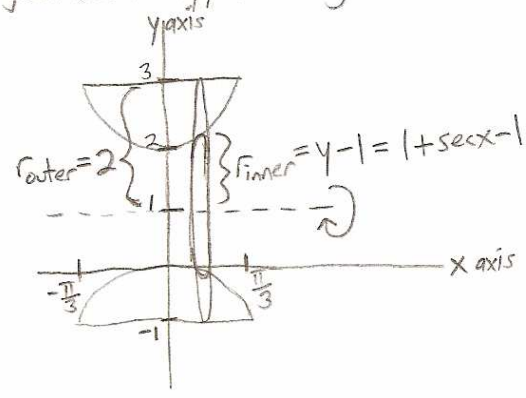
$$\Rightarrow A(x)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[(1-x)^2-(1-\sqrt{x})^2\right]$$
$$=\pi\left[\cancel{1}-2x+x^2-(\cancel{1}-2\sqrt{x}+x)\right]=\pi\left[-3x+x^2+2x^{\frac{1}{2}}\right].$$

$$V=\int_{x_1}^{x_2}A(x)\,dx=\int_0^1\pi\left[-3x+x^2+2x^{\frac{1}{2}}\right]dx=\pi\left[-\frac{3x^2}{2}+\frac{x^3}{3}+2\cdot\frac{2}{3}x^{\frac{3}{2}}\right]_0^1$$
$$=\pi\left[-\frac{3}{2}+\frac{1}{3}+\frac{4}{3}\right]=\pi\left[\frac{-9+2+8}{6}\right]=\boxed{\frac{\pi}{6}}$$

Washer 13. $y=1+\sec x$, $y=3$; about $y=1$. The curves intersect when $1+\sec x=3 \Rightarrow \sec x=2 \Rightarrow \cos x=\frac{1}{2}$
$\Rightarrow x=-\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$, etc.



Let's just use the upper ∪ region between $-\frac{\pi}{3}$ and $\frac{\pi}{3}$:



$r_{inner}=y-1=1+\sec x-1=\sec x.$

$$A(x)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[2^2-\sec^2x\right].$$

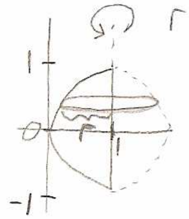
$$V=\int_{x_1}^{x_2}A(x)\,dx=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\pi\left[4-\sec^2x\right]dx=\pi\left[4x-\tan x\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}=\pi\left[\left(\frac{4\pi}{3}-\sqrt{3}\right)-\left(-\frac{4\pi}{3}-(-\sqrt{3})\right)\right]$$

$$=\pi\left[\frac{4\pi}{3}-\sqrt{3}+\frac{4\pi}{3}-\sqrt{3}\right]=\boxed{\pi\left[\frac{8\pi}{3}-2\sqrt{3}\right]}\text{ OR }\boxed{\frac{8\pi^2}{3}-2\pi\sqrt{3}}\text{ OR }\boxed{2\pi\left[\frac{4\pi}{3}-\sqrt{3}\right]}$$

(After this, by symmetry it could be done as $2\int_0^{\frac{\pi}{3}}\pi\left[4-\sec^2x\right]dx\ldots$)

15. $x=y^2$, $x=1$; about $x=1$

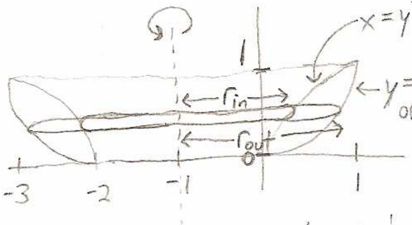
$r=1-x=1-y^2$. $A(y)=\pi r^2=\pi(1-y^2)^2=\pi(1-2y^2+y^4)$.



$$V=\int_{y_1}^{y_2}A(y)\,dy=\int_{-1}^{1}\pi[1-2y^2+y^4]\,dy \underset{\substack{\uparrow\\ \text{By symmetry}}}{=} 2\int_0^1\pi[1-2y^2+y^4]\,dy$$

$$=2\pi\left[y-\tfrac{2}{3}y^3+\tfrac{y^5}{5}\right]_0^1=2\pi\left[1-\tfrac{2}{3}+\tfrac{1}{5}\right]=2\pi\left[\tfrac{15-10+3}{15}\right]=\boxed{\tfrac{16\pi}{15}}$$

17. $y=x^2$, $x=y^2$; about $x=-1$. $r_{inner}=1+x=1+y^2$. $r_{outer}=1+x=1+\sqrt{y}$.



$$A(y)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[(1+\sqrt{y})^2-(1+y^2)^2\right]$$

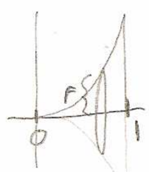
$$=\pi\left[\cancel{1}+2\sqrt{y}+y-(\cancel{1}+2y^2+y^4)\right]$$

$$=\pi\left[2y^{\frac{1}{2}}+y-2y^2-y^4\right].$$

$$V=\int_{y_1}^{y_2}A(y)\,dy=\int_0^1\pi\left[2y^{\frac{1}{2}}+y-2y^2-y^4\right]=\pi\left[2\cdot\tfrac{2}{3}y^{\frac{3}{2}}+\tfrac{y^2}{2}-\tfrac{2}{3}y^3-\tfrac{y^5}{5}\right]_0^1=\pi\left[\tfrac{4}{3}+\tfrac{1}{2}-\tfrac{2}{3}-\tfrac{1}{5}\right]$$

$$=\pi\left[\tfrac{40+15-20-6}{30}\right]=\boxed{\tfrac{29\pi}{30}}$$

19. $y=x^3$, $y=0$, $x=1$; about the x-axis.

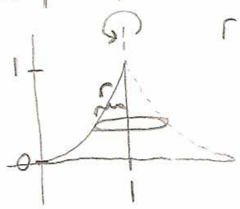


$r=y=x^3$. $A(x)=\pi r^2=\pi(x^3)^2=\pi x^6$.

$$V=\int_{x_1}^{x_2}A(x)\,dx=\int_0^1\pi x^6\,dx=\pi\left[\tfrac{x^7}{7}\right]_0^1=\boxed{\tfrac{\pi}{7}}$$

21. $y=x^3$, $y=0$, $x=1$; about $x=1$.

$r=1-x=1-\sqrt[3]{y}$. $A(y)=\pi r^2=\pi(1-\sqrt[3]{y})^2=\pi(1-2\sqrt[3]{y}+y^{\frac{2}{3}})$.

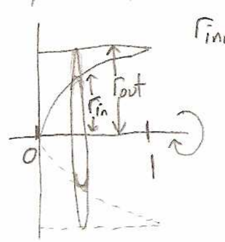


$$V=\int_{y_1}^{y_2}A(y)\,dy=\int_0^1\pi\left[1-2y^{\frac{1}{3}}+y^{\frac{2}{3}}\right]dy=\pi\left[y-2\cdot\tfrac{3}{4}y^{\frac{4}{3}}+\tfrac{3}{5}y^{\frac{5}{3}}\right]_0^1$$

$$=\pi\left[1-\tfrac{3}{2}+\tfrac{3}{5}\right]=\pi\left[\tfrac{10-15+6}{10}\right]=\boxed{\tfrac{\pi}{10}}$$

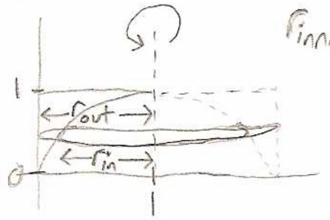
23. $y=\sqrt{x}$, $y=1$, $x=0$; about the x-axis.

$r_{inner}=y=\sqrt{x}$. $r_{outer}=1$. $A(x)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[1^2-(\sqrt{x})^2\right]=\pi[1-x]$.



$$V=\int_{x_1}^{x_2}A(x)\,dx=\int_0^1\pi[1-x]\,dx=\pi\left[x-\tfrac{x^2}{2}\right]_0^1=\pi\left[1-\tfrac{1}{2}\right]=\boxed{\tfrac{\pi}{2}}$$

25. $y=\sqrt{x}$, $y=1$, $x=0$; about $x=1$.



$r_{inner}=1-x=1-y^2$. $r_{outer}=1$. $A(y)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[1^2-(1-y^2)^2\right]$

$=\pi\left[\cancel{1}-(\cancel{1}-2y^2+y^4)\right]=\pi\left[2y^2-y^4\right]$

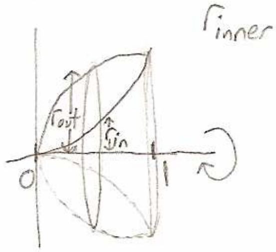
$V=\int_{y_1}^{y_2}A(y)\,dy=\int_0^1\pi\left[2y^2-y^4\right]dy=\pi\left[\frac{2}{3}y^3-\frac{y^5}{5}\right]_0^1=\pi\left[\frac{2}{3}-\frac{1}{5}\right]$

$=\pi\left[\frac{10-3}{15}\right]=\boxed{\frac{7\pi}{15}}$

27. $y=\sqrt{x}$, $y=x^3$; about the x-axis

$r_{inner}=y=x^3$. $r_{outer}=y=\sqrt{x}$. $A(x)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[(\sqrt{x})^2-(x^3)^2\right]$

$=\pi\left[x-x^6\right]$.

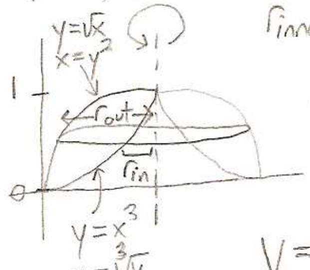


$V=\int_{x_1}^{x_2}A(x)\,dx=\int_0^1\pi\left[x-x^6\right]dx=\pi\left[\frac{x^2}{2}-\frac{x^7}{7}\right]_0^1=\pi\left[\frac{1}{2}-\frac{1}{7}\right]=\pi\left[\frac{7-2}{14}\right]$

$=\boxed{\frac{5\pi}{14}}$

29. $y=\sqrt{x}$, $y=x^3$; about $x=1$.

$r_{inner}=1-x=1-\sqrt[3]{y}$. $r_{outer}=1-x=1-y^2$.



$A(y)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[(1-y^2)^2-(1-\sqrt[3]{y})^2\right]$

$=\pi\left[\cancel{1}-2y^2+y^4-(\cancel{1}-2\sqrt[3]{y}+y^{\frac{2}{3}})\right]=\pi\left[-2y^2+y^4+2y^{\frac{1}{3}}-y^{\frac{2}{3}}\right]$.

$V=\int_{y_1}^{y_2}A(y)\,dy=\int_0^1\pi\left[-2y^2+y^4+2y^{\frac{1}{3}}-y^{\frac{2}{3}}\right]dy=\pi\left[-\frac{2}{3}y^3+\frac{y^5}{5}+2\cdot\frac{3}{4}y^{\frac{4}{3}}-\frac{3}{5}y^{\frac{5}{3}}\right]_0^1$

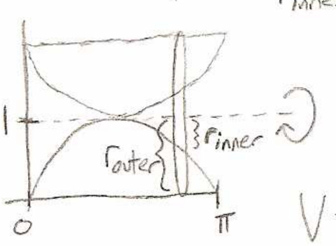
$=\pi\left[-\frac{2}{3}+\frac{1}{5}+\frac{3}{2}-\frac{3}{5}\right]=\pi\left[\frac{-20+6+45-18}{30}\right]=\boxed{\frac{13\pi}{30}}$

33. $y=0$, $y=\sin x$, $0\le x\le\pi$; about $y=1$.

$r_{inner}=1-y=1-\sin x$. $r_{outer}=1$. $A(x)=\pi r_{outer}^2-\pi r_{inner}^2=\pi\left[1^2-(1-\sin x)^2\right]$

$=\pi\left[\cancel{1}-(\cancel{1}-2\sin x+\sin^2 x)\right]$

$=\pi\left[2\sin x-\frac{1}{2}(1-\cos 2x)\right]$.



$V=\int_{x_1}^{x_2}A(x)\,dx=\int_0^{\pi}\pi\left[2\sin x-\frac{1}{2}+\frac{1}{2}\cos 2x\right]dx=\pi\left[-2\cos x-\frac{1}{2}x+\frac{1}{4}\sin 2x\right]_0^{\pi}$

$=\pi\left[\left(-2(-1)-\frac{\pi}{2}+0\right)-(-2-0+0)\right]=\pi\left[2-\frac{\pi}{2}+2\right]=\boxed{\pi\left[4-\frac{\pi}{2}\right]}$

$=\boxed{4\pi-\frac{\pi^2}{2}}$