1. $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$. $\frac{1}{n+3^n} < \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$, and $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$ is a convergent geometric series $(r=\frac{1}{3}<1)$, so $\sum_{n=1}^{\infty}\frac{1}{n+3^n}$ $\boxed{\text{converges}}$ by the Comparison Test.

2. $\sum_{n=1}^{\infty}\frac{(2n+1)^n}{n^{2n}} = \sum_{n=1}^{\infty}\frac{(2n+1)^n}{(n^2)^n} = \sum_{n=1}^{\infty}\left(\frac{2n+1}{n^2}\right)^n$. Using the Root Test:

$\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{2n+1}{n^2}\right)^n} = \lim_{n\to\infty}\frac{2n+1}{n^2} = 0 < 1$, so the series $\boxed{\text{converges}}$ by the Root Test.

3. $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$ $\lim_{n\to\infty}\frac{n}{n+2} = 1$, so $\lim_{n\to\infty}(-1)^n\frac{n}{n+2}$ D.N.E. Therefore, the series $\boxed{\text{diverges}}$ by the Test for Divergence.

4. $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+2}$ $\lim_{n\to\infty}\frac{n}{n^2+2} = 0$, and $\left\{\frac{n}{n^2+2}\right\}$ is positive and decreasing, so by the Alternating Series Test the series $\boxed{\text{converges}}$.

5. $\sum_{n=1}^{\infty}\frac{n^2 2^{n-1}}{(-5)^n}$ Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^2 2^n}{5^{n+1}}\cdot\frac{5^n}{n^2\cdot 2^{n-1}} = \left(\frac{n+1}{n}\right)^2\cdot\frac{2\cdot\cancel{2^{n-1}}}{\cancel{2^{n-1}}}\cdot\frac{\cancel{5^n}}{5\cdot\cancel{5^n}}$

$= \left(1+\frac{1}{n}\right)^2\cdot\frac{2}{5} \to \frac{2}{5} < 1$ as $n\to\infty$, so the series $\boxed{\text{converges}}$ by the Ratio Test.

6. $\sum_{n=1}^{\infty}\frac{1}{2n+1}$ Using the Limit Comparison Test with $a_n = \frac{1}{2n+1}$ and $b_n = \frac{1}{n}$:

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{1}{2n+1}\cdot\frac{n}{1} = \lim_{n\to\infty}\frac{n}{2n+1} = \frac{1}{2}$ (a finite # >0), so since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, by the Limit Comparison Test $\sum_{n=1}^{\infty}\frac{1}{2n+1}$ also $\boxed{\text{diverges}}$.

7. $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$ Integral Test: $\int_2^{\infty}\frac{1}{x\sqrt{\ln x}}\,dx = \lim_{t\to\infty}\left[2\sqrt{\ln x}\right]_2^t = \lim_{t\to\infty}\left[2\sqrt{\ln t} - 2\sqrt{\ln 2}\right]$
$= \infty$

$\int\frac{1}{x\sqrt{\ln x}}\,dx = \int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}$
$= 2\sqrt{\ln x}$

$u = \ln x$
$du = \frac{1}{x}\,dx$

Since the integral diverges, by the Integral Test the series $\boxed{\text{diverges.}}$

8. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!} = \sum_{k=1}^{\infty} \frac{2^k \cdot \cancel{k!}}{(k+2)(k+1)\cancel{k!}} = \sum_{k=1}^{\infty} \frac{2^k}{(k+2)(k+1)}$ Ratio Test:

$\left|\frac{a_{k+1}}{a_k}\right| = \frac{2^{k+1}}{(k+3)\cancel{(k+2)}} \cdot \frac{\cancel{(k+2)}(k+1)}{2^k} = \frac{2 \cdot \cancel{2^k}}{\cancel{2^k}} \cdot \frac{k+1}{k+3} \to 2 \cdot 1 = 2 > 1$ as $n \to \infty$,

so the series $\boxed{\text{diverges}}$ by the Ratio Test.

9. $\sum_{k=1}^{\infty} k^2 e^{-k} = \sum_{k=1}^{\infty} \frac{k^2}{e^k}$ Ratio Test: $\left|\frac{a_{k+1}}{a_k}\right| = \frac{(k+1)^2}{e^{k+1}} \cdot \frac{e^k}{k^2} = \frac{1}{e} \cdot \left(\frac{k+1}{k}\right)^2 \to \frac{1}{e} < 1$, (as $n \to \infty$)

so the series $\boxed{\text{converges}}$ by the Ratio Test.

10. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ Integral Test: $\int_1^{\infty} x^2 e^{-x^3} dx = \lim_{t \to \infty} \left[-\frac{1}{3} e^{-x^3}\right]_1^t = -\frac{1}{3} \lim_{t \to \infty} \left[e^{-t^3} - e^{-1}\right]$

$-\frac{1}{3} \int x^2 e^{-x^3} dx^{(-3)} = -\frac{1}{3} \int e^u du = -\frac{1}{3} e^u = -\frac{1}{3} e^{-x^3}$ $\quad = -\frac{1}{3} \lim_{t \to \infty} \left[\overset{0}{\cancel{\frac{1}{e^{t^3}}}} - \frac{1}{e}\right] = \frac{1}{3e}$.

$u = -x^3$, $du = -3x^2 dx$

Since the integral converges, by the Integral Test the series $\boxed{\text{converges}}$.

11. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$ $\quad \lim_{n \to \infty} \frac{1}{n \ln n} = 0$, and $\left\{\frac{1}{n \ln n}\right\}$ is positive and decreasing, so by the Alternating Series Test the series $\boxed{\text{converges}}$.

12. $\sum_{n=1}^{\infty} \sin n$ $\quad \lim_{n \to \infty} \sin n$ D.N.E., so the series $\boxed{\text{diverges}}$ by the Test for Divergence.

13. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1}(n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n n^2} = \frac{3 \cdot \cancel{3^n}}{\cancel{3^n}} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}} \cdot \left(\frac{n+1}{n}\right)^2$

$= \frac{3}{n+1} \cdot \left(1 + \frac{1}{n}\right)^2 \to 0 < 1$ as $n \to \infty$, so the series $\boxed{\text{converges}}$ by the Ratio Test.

14. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$. $\quad |\sin 2n| \le 1$, so $\left|\frac{\sin 2n}{1+2^n}\right| \le \frac{1}{1+2^n} < \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$.

Since $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges (geometric series, $|r| = \frac{1}{2} < 1$), by the Comparison Test the series $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$ is absolutely convergent and thus $\boxed{\text{Convergent.}}$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{2\cdot 5\cdot 8\cdot \cdots \cdot (3n+2)}$ Ratio Test: $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!}{\cancel{2\cdot 5\cdot 8\cdot \cdots}\cancel{(3n+2)}(3[n+1]+2)}\cdot\frac{\cancel{2\cdot 5\cdot 8\cdot \cdots}\cancel{(3n+2)}}{n!}$

$\displaystyle = \frac{(n+1)\cancel{n!}}{(3n+5)\cancel{n!}} \to \frac{1}{3} < 1$, so the series $\boxed{\text{converges}}$ by the Ratio Test.

16. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ Using the Limit Comparison Test with $a_n = \frac{n^2+1}{n^3+1}$ and $b_n = \frac{1}{n}$:

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2+1}{n^3+1}\cdot\frac{n}{1} = \lim_{n\to\infty} \frac{n^3+n}{n^3+1} = \underbrace{1}_{\text{finite \# > 0}}$, so since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, by the Limit Comparison Test the series $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3+1}$ also must $\boxed{\text{diverge}}$.

17. $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{\frac{1}{n}}$ $\displaystyle\lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1$, so $\displaystyle\lim_{n\to\infty} (-1)^n 2^{\frac{1}{n}}$ D.N.E. Therefore, by the Test for Divergence, the series $\boxed{\text{diverges}}$.

18. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$ $\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}-1} = 0$, and $\left\{\frac{1}{\sqrt{n}-1}\right\}$ is positive and decreasing, so by the Alternating Series Test the series $\boxed{\text{Converges}}$.

19. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ $\displaystyle\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}}$

$\frac{\infty}{\infty}$ Need L'Hospital's Rule

$= 0$, and $\left\{\frac{\ln n}{\sqrt{n}}\right\}$ is positive and decreasing*, so by the Alternating Series Test the series $\boxed{\text{converges}}$.

$\displaystyle * f(x) = \frac{\ln x}{\sqrt{x}}$ $\displaystyle f'(x) = \frac{\sqrt{x}\cdot\frac{1}{x} - \ln x\cdot\frac{1}{2\sqrt{x}}}{x} = \frac{\frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}}}{x} = \frac{2-\ln x}{2\sqrt{x}\cdot x} < 0$ for $2 - \ln x < 0$

$\Leftrightarrow 2 < \ln x$

$\Leftrightarrow \ln x > 2$

$\Leftrightarrow x > e^2.$

The sequence is decreasing for $n > e^2$.

20. $\displaystyle\sum_{k=1}^{\infty} \frac{k+5}{5^k}$ Ratio Test: $\displaystyle\left|\frac{a_{k+1}}{a_k}\right| = \frac{k+6}{5^{k+1}}\cdot\frac{5^k}{k+5} = \frac{1}{5}\cdot\frac{k+6}{k+5} \to \frac{1}{5} < 1$ as $n\to\infty$, so the series $\boxed{\text{converges}}$ by the Ratio Test.

21. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n} = \sum_{n=1}^{\infty} \frac{[(-2)^2]^n}{n^n} = \sum_{n=1}^{\infty} \left(\frac{4}{n}\right)^n$. By the Root Test:

$\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{4}{n}\right)^n} = \lim_{n\to\infty} \frac{4}{n} = 0 < 1$, so the series $\boxed{\text{converges}}$ by the Root Test.
(converges absolutely)

22. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ $\qquad \displaystyle\frac{\sqrt{n^2-1}}{n^3+2n^2+5} < \frac{\sqrt{n^2}}{n^3+2n^2+5} = \frac{n}{n^3+2n^2+5} < \frac{n}{n^3} = \frac{1}{n^2}$.

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series, $p=2>1$), by the Comparison Test the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ $\boxed{\text{converges}}$.

OR: Using the Limit Comparison Test with $b_n = \frac{1}{n^2}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5} \cdot \frac{n^2}{1} \underset{\div \text{ num. and den. by } n^3}{=} \lim_{n\to\infty} \frac{\frac{\sqrt{n^2-1}}{\sqrt{n^2}} \cdot \frac{n^2}{n^2}}{1+\frac{2}{n}+\frac{5}{n^3}} = \lim_{n\to\infty} \frac{\sqrt{1-\frac{1}{n^2}} \cdot 1}{1+\frac{2}{n}+\frac{5}{n^3}} = \underbrace{1}_{\text{finite \# }>0},$$

so since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, by the Limit Comparison Test the original series also must $\boxed{\text{converge}}$.

23. $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ Using the Limit Comparison Test with $b_n = \frac{1}{n}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \underbrace{\frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}}}_{\frac{0}{0} \text{ Need LHospital}} = \lim_{x\to\infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} \overset{H}{=} \lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right) \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \sec^2 0 = 1^2 = \underbrace{1}_{\text{finite \# }>0},$$

so since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, by the Limit Comparison Test $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ also $\boxed{\text{diverges.}}$

24. $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ $\qquad \displaystyle\underbrace{\lim_{n\to\infty} n \sin\left(\frac{1}{n}\right)}_{\infty \cdot 0 \text{ Indeterminate}} = \lim_{x\to\infty} \underbrace{\frac{\sin\frac{1}{x}}{\frac{1}{x}}}_{\frac{0}{0} \text{ Ready for LHospital}} \overset{H}{=} \lim_{x\to\infty} \frac{\cos\frac{1}{x} \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \cos 0 = 1 \neq 0,$

so by the Test for Divergence the series $\boxed{\text{diverges}}$.

25. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$ Ratio Test: $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n!} = \frac{(n+1)\cancel{n!}}{\cancel{n!}} \cdot \frac{e^{n^2}}{e^{n^2+2n+1}}$

$\displaystyle= \frac{(n+1)\cancel{e^{n^2}}}{\cancel{e^{n^2}} \cdot e^{2n+1}}$ $\qquad \displaystyle\lim_{n\to\infty} \underbrace{\frac{n+1}{e^{2n+1}}}_{\frac{\infty}{\infty} \text{ Need LHospital}} = \lim_{x\to\infty} \frac{x+1}{e^{2x+1}} \overset{H}{=} \lim_{x\to\infty} \frac{1}{e^{2x+1} \cdot 2} = 0 < 1$, so by the Ratio Test the series $\boxed{\text{converges}}$.

26. $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$ Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^2+1}{5^{n+1}} \cdot \frac{5^n}{n^2+1} = \frac{n^2+2n+2}{n^2+1} \cdot \frac{1}{5} \to \frac{1}{5} < 1$ as $n \to \infty$.

so the series $\boxed{\text{converges}}$ by the Ratio Test.

27. $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$. $\frac{k \ln k}{(k+1)^3} < \frac{k \ln k}{k^3} = \frac{\ln k}{k^2}$. Using the Integral Test:

$$\int_1^{\infty} \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \ln x \cdot x^{-2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{x}(\ln x+1)\right]_1^t = \lim_{t\to\infty}\left[-\left(\frac{\ln t+1}{t}\right) + \frac{1}{1}\right]$$

$$\overset{H}{=} \lim_{t\to\infty}\left[-\frac{\left(\frac{1}{t}\right)}{1}\right] + 1 = 0+1 = 1.$$

$$\int \ln x \cdot x^{-2}\,dx = -\frac{1}{x}\ln x + \int x^{-2}\,dx = -\frac{1}{x}\ln x + \left(-\frac{1}{x}\right) = -\frac{1}{x}(\ln x+1)$$

$u = \ln x$ $\quad v = -\frac{1}{x}$

$du = \frac{1}{x}dx$ $\quad dv = x^{-2}dx$

Since the integral converges, the series $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$ converges by the Integral Test.

Therefore, by the Comparison Test, the series $\sum_{k=1}^{\infty} \frac{k\ln k}{(k+1)^3}$ $\boxed{\text{converges}}$.

28. $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ $\quad e^{\frac{1}{n}} \le e$, so $\frac{e^{\frac{1}{n}}}{n^2} \le \frac{1}{n^2}$. Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series, $p=2>1$),

by the Comparison Test the series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ $\boxed{\text{converges}}$.

OR: $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx = \lim_{t\to\infty}\left[-e^{\frac{1}{x}}\right]_1^t = -\lim_{t\to\infty}\left[e^{\frac{1}{t}} - e^1\right] = -\left[e^0 - e^1\right] = -[1-e] = e-1.$

$u = \frac{1}{x}$, $du = -\frac{1}{x^2}dx$ $\quad$ Since the integral converges, by the Integral Test the series $\boxed{\text{converges}}$.

$-\int e^u\,du = -e^u = -e^{\frac{1}{x}}$

29. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(\frac{e^n+e^{-n}}{2}\right)} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2}{e^n+e^{-n}}$ $\quad \lim_{n\to\infty} \frac{2}{e^n+e^{-n}} = 0$ (and $\left\{\frac{2}{e^n+e^{-n}}\right\}$

is positive and decreasing), so by the Alternating Series Test the series $\boxed{\text{converges}}$.

OR: $\frac{2}{e^n+e^{-n}} < \frac{2}{e^n}$. $\sum_{n=1}^{\infty} \frac{2}{e^n} = 2\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$ is convergent (geometric, $|r| = \frac{1}{e} < 1$),

so by the Comparison Test the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$ is absolutely convergent and therefore $\boxed{\text{convergent}}$.

30. $\displaystyle\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$ $\displaystyle\lim_{j\to\infty} \frac{\sqrt{j}}{j+5} = 0$ (and $\left\{\frac{\sqrt{j}}{j+5}\right\}$ is positive and decreasing),

so by the Alternating Series Test the series $\boxed{\text{converges}}$.

31. $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$ $\displaystyle\lim_{k\to\infty} \frac{5^k}{3^k+4^k} = \lim_{k\to\infty} \frac{\frac{5^k}{4^k}}{\frac{3^k}{4^k}+\frac{4^k}{4^k}} = \lim_{k\to\infty} \frac{\left(\frac{5}{4}\right)^k}{\left(\frac{3}{4}\right)^k+1} = \frac{\infty}{0+1} = \infty$

Therfore, by the Test for Divergence, the series $\boxed{\text{diverges}}$.

32. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}} = \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^4)^n} = \sum_{n=1}^{\infty} \left(\frac{n!}{n^4}\right)^n$. Root Test: $\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} =$

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n!}{n^4}\right)^n} = \lim_{n\to\infty} \frac{n!}{n^4} = \lim_{n\to\infty} \frac{n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdot(n-4)!}{n\cdot n\cdot n\cdot n} =$$

$\displaystyle\lim_{n\to\infty} 1\cdot\left(1-\frac{1}{n}\right)\cdot\left(1-\frac{2}{n}\right)\cdot\left(1-\frac{3}{n}\right)\cdot(n-4)! = 1\cdot1\cdot1\cdot1\cdot\infty = \infty$, so the series $\boxed{\text{diverges}}$ by the Root Test.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{\sqrt{n}}$ Using the Limit Comparison Test with $b_n = \frac{1}{n^{\frac{3}{2}}}$:

(tricky to see what to compare to)

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\sqrt{n}}\cdot\frac{n^{\frac{3}{2}}}{1} = \lim_{n\to\infty} \sin\frac{1}{n}\cdot n = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \overset{H}{=} \lim_{x\to\infty} \frac{\cos\frac{1}{x}\cdot\left(-\cancel{\frac{1}{x^2}}\right)}{\left(-\cancel{\frac{1}{x^2}}\right)}$$

$\frac{0}{0}$ ready for L'Hospital

$= \cos 0 = 1 \leftarrow$ a finite # >0,

so the series $\boxed{\text{converges}}$ by the Limit Comparison Test.

34. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+n\cos^2 n}$ $\cos^2 n \le 1$, so $n\cos^2 n \le n$, so $n+n\cos^2 n \le n+n = 2n$,

so $\displaystyle\frac{1}{n+n\cos^2 n} \ge \frac{1}{2n}$. Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, by the Comparison Test the original series $\boxed{\text{diverges}}$.

35. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ Root Test: $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{\left(\frac{n^2}{n}\right)}$

$= \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$. Let $y=\left(\frac{n}{n+1}\right)^n$, so $\ln y = n\ln\left(\frac{n}{n+1}\right)$.

1^∞ indeterminate

Now: $\lim_{n\to\infty}\ln y = \lim_{n\to\infty} n\cdot\ln\left(\frac{n}{n+1}\right) = \lim_{x\to\infty}\frac{\ln\left(\frac{x}{x+1}\right)}{\frac{1}{x}} \overset{H}{=} \lim_{x\to\infty}\frac{\frac{\cancel{x+1}}{x}\cdot\frac{x+1-x}{(x+1)^{\cancel{2}}}}{-\frac{1}{x^2}}$

$\frac{0}{0}$ ready for L'Hospital

$= \lim_{x\to\infty}\frac{1}{x(x+1)}\cdot\frac{-x^2}{1} = -\lim_{x\to\infty}\frac{x}{x+1} = -1$. So, $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} y = e^{-1} = \frac{1}{e} < 1$.

Therefore, by the Root Test the series $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ $\boxed{\text{converges}}$.

[OR] Shorter:

From $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$, $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\frac{1}{\left(\frac{n+1}{n}\right)^n} = \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} < 1$.

Therefore, by the Root Test the series $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ $\boxed{\text{converges}}$.

36. $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^{\ln n}}$ In order to use the Comparison Test, the following manipulations are necessary:

trickery required

$(\ln n)^{\ln n} = \left[e^{\ln(\ln n)}\right]^{\ln n} = \left[e^{\ln n}\right]^{\ln(\ln n)} = n^{\ln(\ln n)} > n^2$ for $n > e^{e^2}$,

switch position of exponents

so $\frac{1}{(\ln n)^{\ln n}} < \frac{1}{n^2}$ for sufficiently large n. Since $\sum_{n=2}^{\infty}\frac{1}{n^2}$ converges, by the Comparison Test the series $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^{\ln n}}$ $\boxed{\text{converges}}$.

37. $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$ Root Test: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{(\sqrt[n]{2}-1)^n}$ = next line

$= \lim_{n\to\infty} \left(\sqrt[n]{2}-1\right) = \lim_{n\to\infty} \left(2^{\frac{1}{n}}-1\right) = 2^0 - 1 = 1-1 = 0 < 1.$ Therefore, the series $\boxed{\text{converges}}$ by the Root Test.

38. $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right) = \sum_{n=1}^{\infty} \left(2^{\frac{1}{n}}-1\right)$ Using the Limit Comparison Test with $b_n = \frac{1}{n}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2^{\frac{1}{n}}-1}{\frac{1}{n}} \overset{H}{=} \lim_{x\to\infty} \frac{2^{\frac{1}{x}} \cdot \left(\cancel{-\frac{1}{x^2}}\right)}{\left(\cancel{-\frac{1}{x^2}}\right)} = 2^0 = 1 \leftarrow \text{a finite \#} > 0.$$

$\frac{0}{0}$ ready for LHospital

Since $\sum \frac{1}{n}$ diverges, by the Limit Comparison Test $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)$ also $\boxed{\text{diverges}}$.