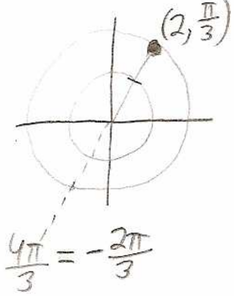
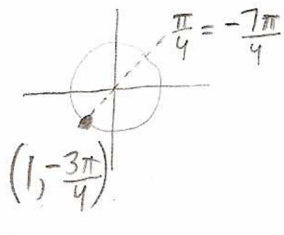
1. Plot the point whose polar coordinates are given. Then find two other pairs of polar coordinates of this point, one with $r>0$ and one with $r<0$.

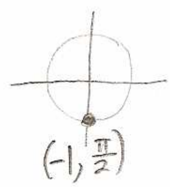
a. $(2, \frac{\pi}{3}) = (2, -\frac{5\pi}{3}) = (2, \frac{7\pi}{3}) = (-2, \frac{4\pi}{3}) = (-2, -\frac{2\pi}{3})$ are some other polar coordinates of the point $(2, \frac{\pi}{3})$.



b. $(1, -\frac{3\pi}{4}) = (1, \frac{5\pi}{4}) = (-1, \frac{\pi}{4}) = (-1, -\frac{7\pi}{4})$ are some others.



c. $(-1, \frac{\pi}{2}) = (1, \frac{3\pi}{2}) = (1, -\frac{\pi}{2}) = (-1, -\frac{3\pi}{2})$
$= (-1, \frac{5\pi}{2})$



3. Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point. (rectangular)

a) $(1, \pi) = \boxed{(-1, 0)}$

r θ x y

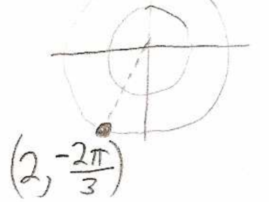
b) $(2, -\frac{2\pi}{3})$

r θ

$x = r\cos\theta = 2\cos(-\frac{2\pi}{3}) = 2(-\frac{1}{2}) = -1$

$y = r\sin\theta = 2\sin(-\frac{2\pi}{3}) = 2(-\frac{\sqrt{3}}{2}) = -\sqrt{3}$

$(x, y) = \boxed{(-1, -\sqrt{3})}$



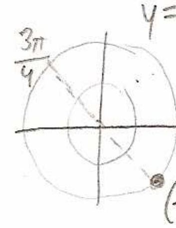
c) $(-2, \frac{3\pi}{4})$

r θ

$x = r\cos\theta = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

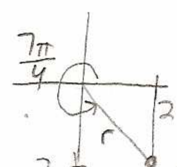
$y = r\sin\theta = 2 \cdot (-\frac{\sqrt{2}}{2}) = -\sqrt{2}$

$(x, y) = \boxed{(\sqrt{2}, -\sqrt{2})}$



5a. The Cartesian coordinates of a point are $(x, y) = (2, -2)$.

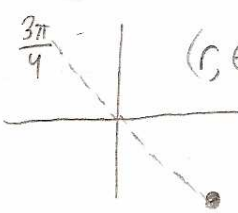
i. Find (r, θ) with $r>0$ and $0 \le \theta < 2\pi$.



$r = \sqrt{x^2+y^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$.

$\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = -1 \Rightarrow \theta = \frac{7\pi}{4}$. $(r, \theta) = \boxed{(2\sqrt{2}, \frac{7\pi}{4})}$

ii. $r<0$:



$(r, \theta) = \boxed{(-2\sqrt{2}, \frac{3\pi}{4})}$

FORMULAS:

$r^2 = x^2 + y^2$

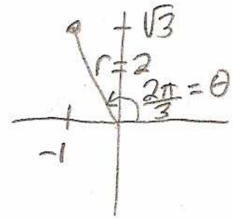
$\tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

5b. The Cartesian coordinates of a point are $(x,y) = (-1, \sqrt{3})$.

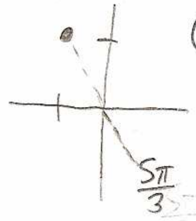
i. Find (r,θ), where $r>0$ and $0 \le \theta < 2\pi$.



$r = \sqrt{x^2+y^2} = \sqrt{1+3} = 2$. $\tan\theta = \frac{y}{x} = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$.

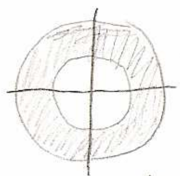
$(r,\theta) = \boxed{\left(2, \frac{2\pi}{3}\right)}$

ii. with $r<0$:



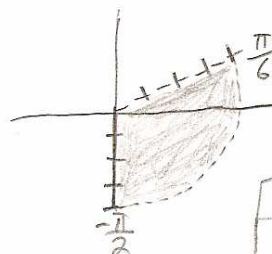
$(r,\theta) = \boxed{\left(-2, \frac{5\pi}{3}\right)}$

7. Sketch the region: $1 \le r \le 2$



all pts. between and on circles of radii 1 and 2 centered at $(0,0)$.

9. Sketch the region: $0 \le r < 4$, $-\frac{\pi}{2} \le \theta < \frac{\pi}{6}$:



FORMULAS:

$r^2 = x^2+y^2$

$\tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

#15-19: Convert the polar equation to a Cartesian equation.

15. $r=2$. Circle, radius 2 center $(0,0) \Rightarrow \boxed{x^2+y^2=4}$.

$r^2=4 \Rightarrow x^2+y^2=4$.

17. $r = 3\sin\theta \Rightarrow r^2 = 3r\sin\theta \Rightarrow x^2+y^2 = 3y \Rightarrow$

Complete the □

$x^2+y^2-3y+\frac{9}{4} = \frac{9}{4} \Rightarrow \boxed{x^2+\left(y-\frac{3}{2}\right)^2 = \frac{9}{4}}$

Circle with center $(0, \frac{3}{2})$, radius $\frac{3}{2}$.

19. $r = \csc\theta \Rightarrow r = \frac{1}{\sin\theta} \Rightarrow r\sin\theta = 1 \Rightarrow \boxed{y=1}$ horizontal line.

#21-25 Convert the Cartesian eq. to a polar equation.

21. $x=3 \Rightarrow r\cos\theta = 3 \Rightarrow r = \frac{3}{\cos\theta} \Rightarrow \boxed{r = 3\sec\theta}$

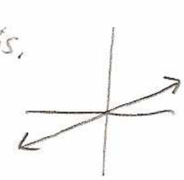
23. $x = -y^2 \Rightarrow r\cos\theta = -r^2\sin^2\theta \Rightarrow \cos\theta = -r\sin^2\theta \Rightarrow r = -\frac{\cos\theta}{\sin^2\theta}$

$\Rightarrow \boxed{r = -\csc\theta\cot\theta}$

25. $x^2+y^2 = 2cx \Rightarrow r^2 = 2cr\cos\theta \Rightarrow \boxed{r = 2c\cos\theta}$

27. a. A line through the origin that makes an angle of $\frac{\pi}{6}$ with the positive x-axis.

$\boxed{\theta = \frac{\pi}{6}}$ or $\tan\theta = \tan\frac{\pi}{6} \Rightarrow \frac{y}{x} = \frac{\sqrt{3}}{3} \Rightarrow \boxed{y = \frac{\sqrt{3}}{3}x}$

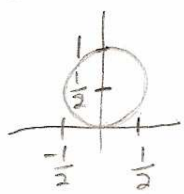


b. a vertical line through the point (3,3). $\boxed{x=3}$ or $r\cos\theta = 3 \Rightarrow \boxed{r = 3\sec\theta}$

#29–43: Sketch the curve.

29. $\theta = -\frac{\pi}{6}$

31. $r = \sin\theta \Rightarrow r^2 = r\sin\theta \Rightarrow x^2+y^2 = y \Rightarrow x^2+y^2-y+\frac{1}{4} = \frac{1}{4} \Rightarrow x^2+(y-\frac{1}{2})^2 = \frac{1}{4}$ circle center $(0,\frac{1}{2})$ radius $\frac{1}{2}$.



θ	r
0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} = .7$
$\frac{\pi}{2}$	1

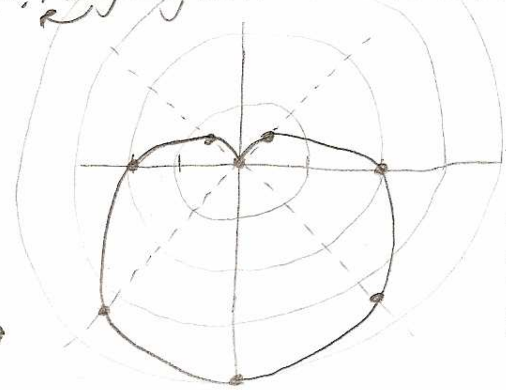
θ	r
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2} = .7$
π	0
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2} = .7$ (beginning 2nd revolution around the circle)

33. $r = 2(1-\sin\theta)$, $\theta \geq 0$

a cardioid

θ	r
0	2
$\frac{\pi}{4}$	$2(.3) = .6$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$2(.3) = .6$
π	2

continued...



θ	r
$\frac{5\pi}{4}$	$2(1.7) = 3.4$
$\frac{3\pi}{2}$	4
$\frac{7\pi}{4}$	$2(1.7) = 3.4$
2π	2

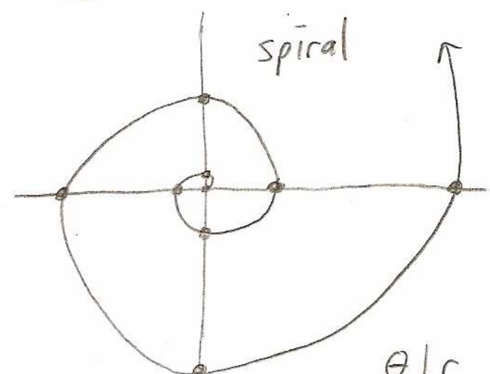
35. $r = \theta$, $\theta \geq 0$

θ	r
0	0
$\frac{\pi}{2}$	1.57
π	3.14
$\frac{3\pi}{2}$	4.71

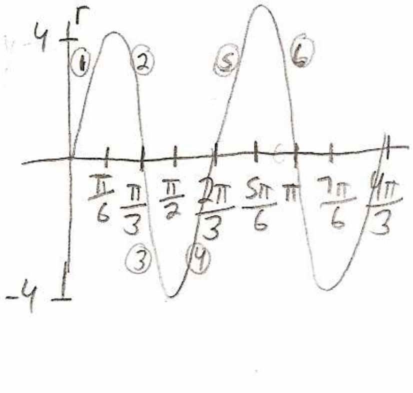
θ	r
2π	6.28
$\frac{5\pi}{2}$	7.85
3π	9.42
$\frac{7\pi}{2}$	11

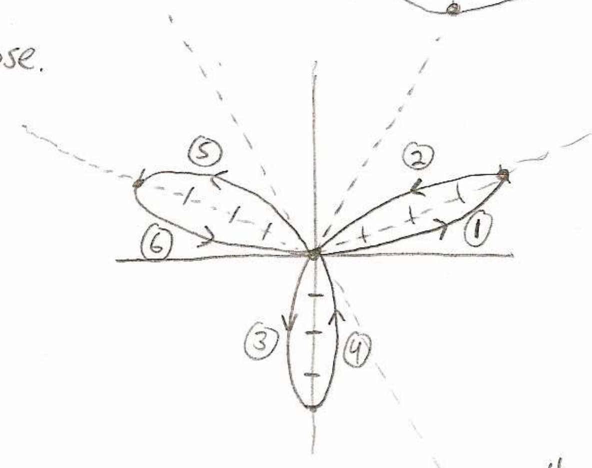
θ	r
4π	12.57

spiral



37. $r = 4\sin 3\theta$ 3 leaved rose.



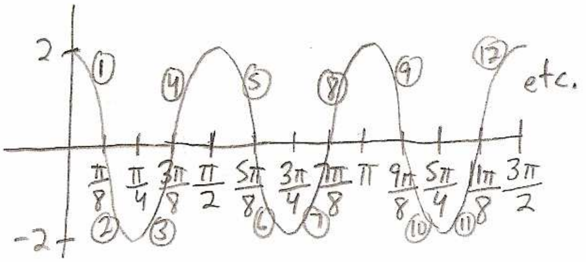


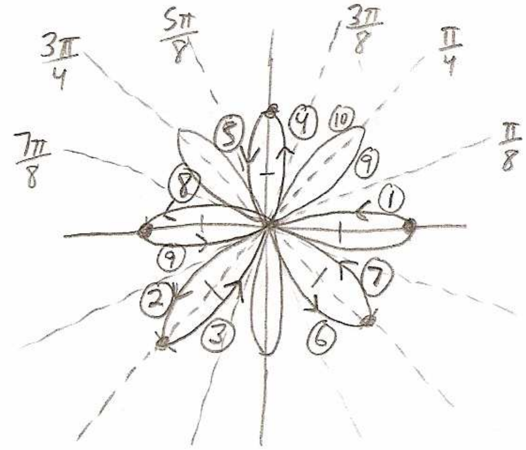
θ	r
0	0
$\frac{\pi}{6}$	4
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	-4
$\frac{2\pi}{3}$	0
$\frac{5\pi}{6}$	4
π	0

then it traces out the same curve again.

39. $r = 2\cos 4\theta$

8-leaved rose



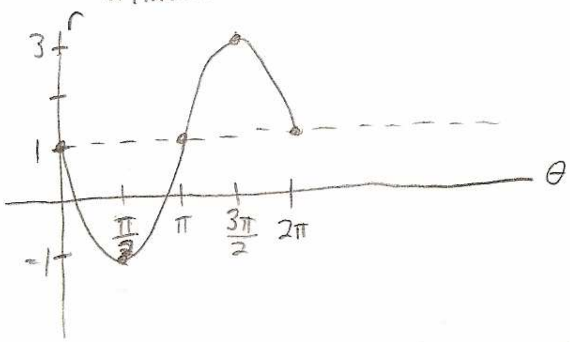


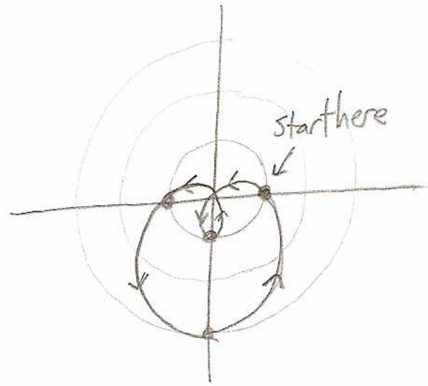
θ	r
0	2
$\frac{\pi}{8}$	0
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	0
$\frac{\pi}{2}$	2
$\frac{5\pi}{8}$	0

etc.

41. $r = 1 - 2\sin\theta = -2\sin\theta + 1$

a limacon with a loop





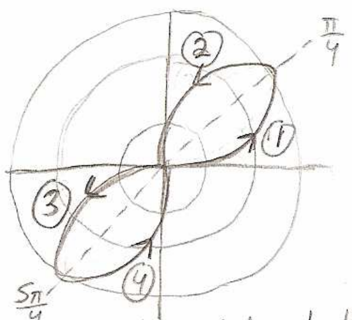
θ	r
0	1
$\frac{\pi}{2}$	-1
π	1
$\frac{3\pi}{2}$	3
2π	1

43. $r^2 = 9\sin 2\theta \longrightarrow r = \sqrt{9\sin 2\theta}$ → $\sin 2\theta \geq 0 \Leftrightarrow 0 \leq 2\theta \leq \pi$ or $2\pi \leq 2\theta \leq 3\pi$

a lemniscate (figure 8) $\quad r = 3\sqrt{\sin 2\theta}$ $\quad 0 \leq \theta \leq \frac{\pi}{2}$ OR $\pi \leq \theta \leq \frac{3\pi}{2}$

θ	r
0	0
$\frac{\pi}{4}$	3
$\frac{\pi}{2}$	0

θ	r
π	0
$\frac{5\pi}{4}$	3
$\frac{3\pi}{2}$	0



#57–61: Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

57. $r = 2\sin\theta, \ \theta = \frac{\pi}{6}$

$x = r\cos\theta = 2\sin\theta\cos\theta = \sin 2\theta$

$y = r\sin\theta = 2\sin\theta\sin\theta = 2\sin^2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{4\sin\theta\cos\theta}{2\cos 2\theta}.$$

When $\theta = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{4\sin\frac{\pi}{6}\cos\frac{\pi}{6}}{2\cos\frac{\pi}{3}} = \frac{4\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{2\cdot\frac{1}{2}} = \boxed{\sqrt{3}.}$ ← slope of the tangent line to the curve $r = 2\sin\theta$ when $\theta = \frac{\pi}{6}$.

59. $r = \frac{1}{\theta}$, $\theta = \pi$

slope of the tangent line when $\theta = \pi$

$x = r\cos\theta = \frac{1}{\theta}\cos\theta = \frac{\cos\theta}{\theta}$

$y = r\sin\theta = \frac{1}{\theta}\sin\theta = \frac{\sin\theta}{\theta}$

When $\theta = \pi$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{\theta\cos\theta - \sin\theta}{\cancel{\theta^2}}}{\frac{\theta(-\sin\theta) - \cos\theta}{\cancel{\theta^2}}} = \frac{\theta\cos\theta - \sin\theta}{-\theta\sin\theta - \cos\theta} = \frac{\pi(-1) - 0}{-\pi\cdot 0 - (-1)} = \frac{-\pi}{1} = \boxed{-\pi}$$

Quotient rule for both derivatives

61. $r = \cos 2\theta$, $\theta = \frac{\pi}{4}$

$x = r\cos\theta = \cos 2\theta \cos\theta$

$y = r\sin\theta = \cos 2\theta \sin\theta$

When $\theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos 2\theta\cdot\cos\theta + (-2\sin 2\theta)\sin\theta}{\cos 2\theta(-\sin\theta) + (-2\sin 2\theta)\cos\theta} = \frac{\cos\frac{\pi}{2}\cdot\cos\frac{\pi}{4} - 2\sin\frac{\pi}{2}\sin\frac{\pi}{4}}{-\cos\frac{\pi}{2}\sin\frac{\pi}{4} - 2\sin\frac{\pi}{2}\cos\frac{\pi}{4}}$$

Product Rule for both derivatives

$$= \frac{0 - \cancel{2}\cdot 1\cdot\frac{\sqrt{2}}{\cancel{2}}}{0 - \cancel{2}\cdot 1\cdot\frac{\sqrt{2}}{\cancel{2}}} = \frac{-\sqrt{2}}{-\sqrt{2}} = \boxed{1}$$

slope of tan. line when $\theta = \frac{\pi}{4}$

#63–67: Find the points on the curve where the tangent line is horizontal or vertical.

63. $r = 3\cos\theta$

$x = r\cos\theta = 3\cos\theta\cos\theta = 3\cos^2\theta$

$y = r\sin\theta = 3\cos\theta\sin\theta = \frac{3}{2}\cdot 2\sin\theta\cos\theta = \frac{3}{2}\sin 2\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{3}{\cancel{2}}\cdot\cancel{2}\cos 2\theta}{3\cdot 2\cos\theta(-\sin\theta)} = \frac{3\cos 2\theta}{-3\cdot 2\sin\theta\cos\theta} = \frac{3\cos 2\theta}{-3\sin 2\theta} \left(= -\frac{\cos 2\theta}{\sin 2\theta}\right)$$

$$\frac{dy}{d\theta} = 0 \text{ when } \cos 2\theta = 0 \Rightarrow \left.\begin{array}{l} 2\theta = \frac{\pi}{2} + 2n\pi \\ 2\theta = \frac{3\pi}{2} + 2n\pi \end{array}\right\} \Rightarrow 2\theta = \frac{\pi}{2} + n\pi \Rightarrow \theta = \frac{\pi}{4} + \frac{\pi}{2}n$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$$

$$\frac{dx}{d\theta} = 0 \text{ when } \sin 2\theta = 0 \Rightarrow \left.\begin{array}{l} 2\theta = 0 + 2n\pi \\ 2\theta = \pi + 2n\pi \end{array}\right\} \Rightarrow 2\theta = n\pi \Rightarrow \theta = \frac{\pi}{2}n = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$$

The tangent line is horizontal when $\frac{dy}{d\theta} = 0$, so substitute $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$ into the equation $r = 3\cos\theta$ to get the points... (next page)

63. continued

θ	r
$\frac{\pi}{4}$	$3\cos\frac{\pi}{4} = 3\cdot\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$
$\frac{3\pi}{4}$	$3\cos\frac{3\pi}{4} = 3\left(-\frac{\sqrt{2}}{2}\right) = -\frac{3\sqrt{2}}{2}$
$\frac{5\pi}{4}$	$3\cos\frac{5\pi}{4} = 3\left(-\frac{\sqrt{2}}{2}\right) = \frac{-3\sqrt{2}}{2}$
$\frac{7\pi}{4}$	$3\cos\frac{7\pi}{4} = 3\cdot\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$

The points (r,θ) where the tangent line is horizontal are $\boxed{\left(\frac{3\sqrt{2}}{2}, \frac{\pi}{4}\right), \left(-\frac{3\sqrt{2}}{2}, \frac{3\pi}{4}\right)}$

[we don't need to list the point $\left(-\frac{3\sqrt{2}}{2}, \frac{5\pi}{4}\right)$ since it is equivalent to $\left(\frac{3\sqrt{2}}{2}, \frac{\pi}{4}\right)$, and we don't need to list the point $\left(\frac{3\sqrt{2}}{2}, \frac{7\pi}{4}\right)$ since it is equivalent to $\left(-\frac{3\sqrt{2}}{2}, \frac{3\pi}{4}\right)$.]

The tangent line is vertical when $\frac{dx}{d\theta} = 0$, so substitute $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$:

θ	r
0	$3\cos 0 = 3\cdot 1 = 3$
$\frac{\pi}{2}$	$3\cdot\cos\frac{\pi}{2} = 3\cdot 0 = 0$
π	$3\cos\pi = 3(-1) = -3$
$\frac{3\pi}{2}$	$3\cos\frac{3\pi}{2} = 3\cdot 0 = 0$

The points (r,θ) where the tangent line is vertical are $\boxed{(3,0) \text{ and } \left(0,\frac{\pi}{2}\right)}$

[the point $(-3,\pi)$ is the same point as $(3,0)$, and $\left(0,\frac{3\pi}{2}\right)$ is the same as $\left(0,\frac{\pi}{2}\right)$, so we don't need to list them.]

65. $r = 1+\cos\theta$

$x = r\cos\theta = (1+\cos\theta)\cos\theta = \cos\theta + \cos^2\theta$

$y = r\sin\theta = (1+\cos\theta)\sin\theta$

$\frac{dx}{d\theta} = -\sin\theta + 2\cos\theta(-\sin\theta) = -\sin\theta(1+2\cos\theta)$. $\frac{dx}{d\theta} = 0 \Leftrightarrow \sin\theta = 0$ or $1+2\cos\theta = 0$

$\theta = 0, \pi, \ldots$ $\quad \cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$

$\frac{dy}{d\theta} = (1+\cos\theta)\cos\theta + (-\sin\theta)\sin\theta = \cos\theta + \cos^2\theta - \sin^2\theta$

$= \cos\theta + \cos^2\theta - (1-\cos^2\theta) = \cos\theta + 2\cos^2\theta - 1 = 2\cos^2\theta + \cos\theta - 1$

$= (2\cos\theta - 1)(\cos\theta + 1)$. $\frac{dy}{d\theta} = 0 \Leftrightarrow \cos\theta = \frac{1}{2}$ OR $\cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ $\quad \theta = \pi, \ldots$

Since both $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ are 0 when $\theta = \pi$, we need to do a limit calculation:

$$\lim_{\theta\to\pi^-} \frac{dy}{dx} = \lim_{\theta\to\pi^-} \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \lim_{\theta\to\pi^-} \frac{(2\cos\theta-1)(\cos\theta+1)}{-\sin\theta(1+2\cos\theta)} = \lim_{\theta\to\pi^-} \frac{\overbrace{2\cos\theta - 1}^{-3}}{\underbrace{1+2\cos\theta}_{-1}} \cdot \frac{\overbrace{\cos\theta+1}^{0}}{\underbrace{\sin\theta}_{0}}$$

$$= 3\cdot\lim_{\theta\to\pi^-} \frac{\cos\theta+1}{\sin\theta} \overset{(H)}{=} 3\cdot\lim_{\theta\to\pi^-} \frac{-\sin\theta}{\cos\theta} = 3\cdot\frac{0}{-1} = 0.$$

Since $\frac{dy}{dx} = 0$ when $\theta = \pi$, there is a horizontal tangent when $\theta = \pi$.

(continued next page...)

So, when $\theta = 0, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$ (these were where $\frac{dx}{dt}=0$) there is a vertical tangent:

θ	r
0	$1+\cos 0 = 1+1 = 2$
$\frac{2\pi}{3}$	$1+\cos\frac{2\pi}{3} = 1-\frac{1}{2} = \frac{1}{2}$
$\frac{4\pi}{3}$	$1+\cos\frac{4\pi}{3} = 1-\frac{1}{2} = \frac{1}{2}$

The points where the tangent line is vertical are

$$\boxed{(2,0),\ \left(\tfrac{1}{2}, \tfrac{2\pi}{3}\right), \text{ and } \left(\tfrac{1}{2}, \tfrac{4\pi}{3}\right)}$$

And, when $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$, and π (these were where $\frac{dy}{dt}=0$) there is a horizontal tangent:

θ	r
$\frac{\pi}{3}$	$1+\cos\frac{\pi}{3} = 1+\frac{1}{2} = \frac{3}{2}$
π	$1+\cos\pi = 1-1 = 0$
$\frac{5\pi}{3}$	$1+\cos\frac{5\pi}{3} = 1+\frac{1}{2} = \frac{3}{2}$

The points where the tangent line is horizontal are

$$\boxed{\left(\tfrac{3}{2}, \tfrac{\pi}{3}\right),\ (0,\pi), \text{ and } \left(\tfrac{3}{2}, \tfrac{5\pi}{3}\right)}$$

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67. $r = 2+\sin\theta$

$x = r\cos\theta = (2+\sin\theta)\cos\theta$

$y = r\sin\theta = (2+\sin\theta)\sin\theta = 2\sin\theta + \sin^2\theta$

$$\frac{dx}{d\theta} = (2+\sin\theta)(-\sin\theta) + \cos\theta\cdot\cos\theta = -2\sin\theta - \sin^2\theta + \cos^2\theta = -2\sin\theta - \sin^2\theta + (1-\sin^2\theta)$$

$= 1 - 2\sin\theta - 2\sin^2\theta$ doesn't factor; must use quadratic formula

$= -2\sin^2\theta - 2\sin\theta + 1$ $[a=2, b=-2, c=1]$ $\frac{dx}{dt}=0 \Rightarrow \sin\theta = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\Rightarrow$ next line

$$\sin\theta = \frac{2\pm\sqrt{4-4\cdot(-2)\cdot 1}}{2(-2)} = \frac{2\pm\sqrt{4+8}}{-4} = \frac{2\pm\sqrt{12}}{-4} = \frac{2\pm 2\sqrt{3}}{-4} = \frac{1\pm\sqrt{3}}{-2} = -\frac{1}{2}\pm\frac{\sqrt{3}}{2}$$

$\sin\theta = -\frac{1}{2}+\frac{\sqrt{3}}{2}$ OR ~~$\sin\theta = -\frac{1}{2}-\frac{\sqrt{3}}{2}$~~ $\leftarrow$ impossible since $-\frac{1}{2}-\frac{\sqrt{3}}{2} < -1$

$\theta_1 = \underbrace{\sin^{-1}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)}_{\approx 21.5^\circ}$ and $\theta_2 = \underbrace{\pi - \theta_1}_{\approx 158.5^\circ}$.

θ	r
$\theta_1 = \sin^{-1}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$	$2+\sin\left(\sin^{-1}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right) = 2+-\frac{1}{2}+\frac{\sqrt{3}}{2} = \frac{3}{2}+\frac{\sqrt{3}}{2}$
$\theta_2 = \pi - \sin^{-1}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$	$2+\sin(\pi-\theta_1) = 2+\sin\theta_1 = \frac{3}{2}+\frac{\sqrt{3}}{2}$ (same as 1st r)

$\sin(\pi - x) = \sin x$

From the table at the right, there are vertical tangent lines at the points $\boxed{\left(\frac{3}{2}+\frac{\sqrt{3}}{2}, \theta_1\right) \text{ and } \left(\frac{3}{2}+\frac{\sqrt{3}}{2}, \theta_2\right)}$

continued next page...

67. Continued... $y = 2\sin\theta + \sin^2\theta$

$$\frac{dy}{d\theta} = 2\cos\theta + 2\sin\theta\cos\theta = 2\cos\theta(1+\sin\theta) = 0 \Rightarrow \cos\theta = 0 \text{ or } \sin\theta = -1$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \theta = \frac{3\pi}{2}$$

θ	r
$\frac{\pi}{2}$	$2+\sin\frac{\pi}{2} = 2+1 = 3$
$\frac{3\pi}{2}$	$2+\sin\frac{3\pi}{2} = 2-1 = 1$

There are horizontal tangent lines at the points $\boxed{\left(3, \frac{\pi}{2}\right) \text{ and } \left(1, \frac{3\pi}{2}\right)}$